BEFORE THE

PUBLIC UTILITY COMMISSION OF OREGON

NW Natural

Rebuttal Testimony of Barbara Summers

UM 1744
Carbon Emission Reduction Program
Combined Heat & Power (CHP)
Cost Recovery

October 16, 2015

Table of Contents

Rates & Regulatory Affairs
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220 N.W. Second Avenue
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I. INTRODUCTION

Q. Please state your name.

A. My name is Barbara Summers.

Q. Are you the same Barbara Summers who previously submitted direct testimony and reply testimony in this proceeding?

A. Yes, my title address, and job responsibilities with Northwest Natural Gas Company (NW Natural or the Company) have not changed.

Q. What is the purpose of your testimony?

A. The purpose of my rebuttal testimony is to respond to new analysis of the Combined Heat and Power (CHP) Solicitation Program (CHP Program) submitted by Staff in its Reply Testimony, Staff/300 and Staff/400. Specifically, I will respond to: 1) the criteria for choosing a carbon reduction calculation methodology; 2) the new analysis of simple payback vs. internal rate of return; 3) the new analysis of the customer incentive; and 4) Staff's argument for NW Natural to share in the risk of the program. I will also provide an update regarding the Environmental Protection Agency's (EPA) eGRID, which recently released new eGRID numbers on October 8, 2015. Based on the new eGRID numbers, Washington State University (WSU) re-ran the RELCOST model for the CHP Program and the results are included in my rebuttal testimony.

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II. CARBON REDUCTION CALCULATION METHODOLOGY

Q: Do you have an update to your testimony regarding EPA's eGRID model?

A. Yes, I do. On October 8, 2015, the EPA released its new version of the eGRID model. The current eGRID numbers are attached in Exhibit NWN/501. For the Company's subregion, the Northwest Power Pool (NWPP) eGRID numbers have decreased from 842 CO_2 lbs./MWh to 655 CO_2 lbs./MWh for baseload emissions and increased from 1,340 CO_2 lbs./MWh to 1,579 CO_2 lbs./MWh for non-baseload emissions. It does not mean that more fossil fules are being added to the portfolio; rather, it means that more fossil fules are being dispatched to meet non-baseload.

Q. How will the increase from 1,340 CO_2 lbs./MWh to 1,579 CO_2 lbs./MWh of the non-baseload value affect the program?

A. NW Natural requested WSU to remodel all prototypes with the 2012 EPA eGRID numbers. Based on that analysis described below, NW Natural is recommending no additional changes to the CHP Program based on the update.

Q. Before knowing about the latest update to the eGRID model, Staff questioned the use of the eGRID model for calculating the GHG emission reductions from the CHP Program. What are Staff's concerns with NW Natural's proposal?

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A. Staff analyzed the four primary methodologies available for calculating the GHG benefits of CHP and recommended that the Northwest Power and Conservation Council (NWPCC) methodology be used.

Q: Does NW Natural agree with the analysis performed by the Commission Staff?

A. No. While the Staff analysis provides adequate criteria for choosing a carbon reduction calculation methodology, NW Natural would evaluate the criteria differently. The chart below from Staff's testimony is provided again for reference:

Emission Reduction Model Criteria

Criertia

	Geographic Inclusion	Frequency of updates	Purpose of Methodology	Transparency	Broad Market Support
eGrid	Too Broad, and far reaching	Data year set is not contemporary			
NWPCC					
ODOE				Staff has not found the data set to be readily available	This methodology has not gained much support
Utility Emission Models				Staff has concerns over the transparency of	Currently unknown

The Company disagrees with Staff's "Transparency" evaluation. Staff states that that "[the NWPCC model] may not grant the kind of accessibility that the EPA eGRID model does…[as it] does use a propriety licensed model which is not easily understandable or accessible to those stakeholders uninitiated to complex dispatch modeling." (*Staff/300, Klotz/18*). eGRID, on the other hand, is a publically available model that can be easily reviewed by all stakeholders. Despite this major difference, Staff scores the NWPCC model and eGRID as equally transparent. NW Natural would change the NWPCC's score for Transparency to "concern that the methodology challenges the criteria but may still present merit" (Yellow). (*Staff/300, Klotz/21*).

Second, for the "Frequency of Updates" criterion, Staff notes the NWPCC updates their model roughly every five years, further explaining "[F]or the purposes of the NW Natural's CHP proposal this interval may be sufficient." (*Staff/300, Klotz/18*). The NWPCC data is given a yellow (or "concern but presents merit"). Staff originally evaluated the EPA data a score of "does not meet the stated criteria" (red). However, the prior eGRID update was released in 2014, and the latest update was made October 8, 2015. NW Natural believes that if NWPCC updates every five years show "concern," than the frequent updates to the eGRID numbers demonstrate that eGRID meets the criteria (green).

Q: Staff believes that the geographic region selected by EPA's CHP Partnership for the specific purpose of calculating displaced emissions from CHP is "too far reaching." Staff states that "[t]his model incorporates plants in Arizona, Utah, Wyoming, and Colorado that do not serve load in Oregon." (*Staff/300, Klotz/12*). Do you agree?

A: No. States outside the Pacific Northwest should also be included in models estimating carbon reductions. The boundaries of the Northwest power system

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are porous – over the course of a year, the region imports and exports power in large quantities, including coal and natural gas generation.

The NWPCC fully acknowledges the importance and relevance of the broader regional system. For example, the Sixth Northwest Conservation and Electric Power Plan (Sixth Plan) clearly states that "the Northwest transmission system is closely integrated into the overall western system".[1] Indeed, the accompanying map in the Sixth Plan, figure 7.1, shows major transmission infrastructure covering a region that is actually *larger* than the NWPP eGRID subregion.

PacifiCorp is an excellent example. Eighty percent of PacifiCorp's energy is natural gas and coal, including marginal gas peaking units.[2] The majority of these natural gas and coal facilities are located in the states of Utah, Wyoming, Colorado, and Arizona. At least a portion of the carbon emissions associated with new CHP in Oregon will have the practical result of reducing gas fired generation and carbon emissions in states outside of the Northwest.

Q: Does EPA speak to why it believes a larger region makes sense when calculating GHG benefits in the electric grid?

[1] http://www.nwcouncil.org/media/6284/SixthPowerPlan.pdf, p. 7-2.

[2] http://www.oregon.gov/energy/pages/oregons_electric_power_mix.aspx

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A. Yes. In its recently released Preamble to the Clean Power Plan (CPP), EPA argues that the larger regions should be used when determining GHG benefits under the CPP. Specifically the agency states:

> We concluded that, absent a compelling reason to adopt a smaller regional scale for evaluation of CO2 emission reduction opportunities for the electric power sector –- which we have not found, as discussed below –- the interconnections should be the regions used for evaluation of the [best system of emissions reduction] for CO2 emission reductions from the electric power sector because of the fundamental characteristics of electricity, the industry's basic interconnected physical infrastructure, and the interdependence of the affected EGUs within each interconnection.[3]

An effort to draw conclusions regarding emissions from a more narrow set of resources – as suggested by Staff – appears to be counter to the policy direction indicated by EPA that favors a much broader regional approach.

Q: Would you recommend adding a criterion to the Staff's evaluation?

A. Yes. The Company would add: "Is the measure currently available to be used." On this measure, EPA's eGRID should score as "green" or "meets." The NWPCC would not meet this criterion. Staff explains the process going forward would require the Council staff to begin work on this measure "beginning after the publication of the Final 7th Power Plan". (*Staff/300, Klotz/22*). The Plan is expected to be final in the January/February timeframe, if there are no delays. Staff's testimony does not provide a clear path forward as to when the measure

[3] 40 CFR Part 60, Carbon Pollution Emissions Guidelines for Existing Stationary Sources: Electric Utility Generating Units, p. 396.

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will be ready, if a different carbon calculation will be required for each and every CHP proposal, and the time it might take for this process to actually result in a CHP project implemented in the state.

Q: How would NW Natural develop a similar scoresheet to the one used by Staff?

A. Based on the foregoing reasons, NW Natural would evaluate the criteria as follows:

Emission Reduction Model						
Criteria						
	Geographic Inclusion	Frequency of Updates	Currently Available	Purpose of Methodology	Transparency	Broad Market Support
eGRID	Broader coverage, more consistent with transmission system	Data set now updated with future updates every 1-2 years				
NWPCC	Narrower coverage linked more to Oregon, less consistent with transmission system	Data set updates every 5 Years	Not now available	Designed for EE could work for CHP	Proprietary model not easily understood	
ODOE				Not developed for CHP	Dataset is not readily available	Methodology has not gained much support
Utility Emission Models			Published in IRP filings	Not developed for CHP	Concerns over transparency	Currently unknown

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III. SIMPLE PAYBACK VS. INTERNAL RATE OF RETURN (IRR)

Q. Please summarize Staff's concerns with the Company's use of simple payback.

A. Staff states that the Company's use of the simple payback methodology to determine the customer incentive level "confuses and distracts from the traditional regulatory standard which is to allow the utility an opportunity to earn its authorized return." (*Staff/400*, *St. Brown/10*). In the alternative, Staff recommends using the IRR methodology to evaluate the appropriate incentive level for CHP Program participants. (*Staff/400, St. Brown/10*).

Q. Does the Company have concerns with Staff's analysis regarding the use of the IRR methodology?

A. No. IRR and both simple and discounted payback are calculated in the WSU model and have been relied on by NWN in determining its recommended customer incentive level. The IRRs and Payback are summarized in the Tables regarding the "Basecase" and "Technical Potential" in the Customer Incentive section of my testimony below. We are, however, concerned with the Staff's application of IRR to our CHP Program. In Staff's Reply Testimony, Staff relies on the book "Investment Analysis and Portfolio Management," Chapter 13 – "*Computing Bond Yields*" to provide the criteria for investment decisions using

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IRR under the CHP Program. (*Staff/400, St. Brown/8*; *Staff/401, St. Brown/5*).[4] The Company believes that the decision for a business to invest in CHP is far more complex than a decision for an investor to invest in a bond. A company will not simply "compare the discount rate . . . to your cost of capital, and accept any investment proposal with an IRR equal to or greater than your cost of capital." (*Staff/400, St. Brown/8*). Staff's assertion that all projects that exceed a company's IRR are adequately incented, does not recognize the capital allocation process of most private and public entities. Most organizations have more potential investments than available capital. The Pew Center on Global Climate Change and ICF International's Survey of Corporate Energy Efficiency Strategies (Pew Survey), states that the "need for capital to pay for projects was the greatest single ongoing challenge, outnumbering any other single item by a four-to-one ratio."[5] (*NWN/503, Summers/11*).

The IRRs are not the same for all projects based on the uncertainty of future cash flows, especially for investments that are not typical to the company/industry or reasonably priced into the Company's cost of capital. Risk is an important component for an IRR and different projects will have different risks associated with them.

Q. Are there other concerns with Staff's analysis of the IRR methodology?

[4] I have provided the entire Chapter 13 as Exhibit NWN/502.

[5] I have provided the entire Pew Survey as Exhibit NWN/503.

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A. Yes. Staff relies on an abstract from the Pew Survey, which states: "Respondents IRR criteria [for investing in energy efficiency] were mostly in the 10-15% range, though one reported a 35% IRR threshold." (*Staff/400, St. Brown/15*; *Staff/401, St. Brown/14*). Staff then uses the IRR range of 10-15% for energy efficiency from the Pew Survey to argue that the proposed payback under the CHP Program is excessive. (*Staff/400, St. Brown/16*).

Q. What is your concern with the above statements and analysis?

A. First, the Pew Survey does not represent a fair data sample to compare to the CHP Program. The Pew Survey is based on 48 companies, ranging in size from $8 billion to $99 billion in revenues with "demonstrated commitment to climate and energy issues." (*NWN/503, Summers/1-2*). The survey states that it "deliberately sought larger companies with strong energy/climate commitments, because the goal is to elicit best practices, not average practices. *In this sense, the sample is intentionally not representative of the U.S. corporate population.*" (*NWN/503, Summers/2*)(emphasis added).

Second, Staff seems to ignore that the Pew Survey equally supports a simple payback methodology. The Pew Survey reports that 91% of the respondents use a standard financial criterion to assess energy efficiency projects, and that simple payback and internal rate of return were the most common criteria. (*NWN/503, Summers/7*). Six of the 15 companies that provided their simple payback period identified a payback period of three years.

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Even though simple payback appears to be commonly accepted in industry, Staff discounted the Company's use of a simple payback as "confus[ing] and distract[ing]." (*Staff/400*, *St. Brown/10*).

Third, only 10 out of 48 companies responded to the survey question that asked for the participating companies' IRR figures. Of the 10 companies, 5 reported IRRs of 10-15%. The other 5 companies reported IRR figures of 18%, 20%, 22%, 25%, and 35%. The remaining companies' IRR figures are unknown. As such, Staff's reliance on the IRR range of 10-15% seems to be questionable given its reliance on such a small sample size and that half of the respondents provided higher IRRs.

Fourth, the Pew Survey is generally geared towards low risk energy efficiency investments. It is not specific to CHP, and does not take into account its additional risks and obstacles. On the other hand, Primen's 2003 Distributive Energy Market Study,[6] which is relied upon in ICF International's Assesment of Technical and Economic Potential for CHP In Oregon,[7] conducted in-depth interviews with 100 managers and executives at companies that had existing CHP systems or a strong interest in acquiring such systems in the 10 MW range. Then, surveyed another 806 businesses; 406 Mass Market businesses (10 kW to

[6] Available at http://www.psc.state.ut.us/utilities/misc/06docs/0699903/0699903TCdocs/Appendix%20A%20to%20Distributed%20Energy%20Report.pdf

[7] NWN/101, Summers 51, Appendix E.

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299 kW demand) and 400 th Large businesses (300 kW to 10 MW demand), again with questions targeted specifically at CHP, to include:

- 130 surveys with Manufacturing companies
- 115 surveys with Schools, Colleges, & Universities;
- 100 with Restaurant and
- 461 with a mix of other SIC categories (excluding agriculture, mining, and construction).

IV. <u>CUSTOMER INCENTIVE</u>

Q. Staff has readdressed issues with the customer incentive level. Could you summarize Staff's concerns?

A. Staff states that customers would participate in the program if the customer incentive was less than $30 per metric ton of carbon dioxide equivalent (MTCO2(e)) of emissions reduction because: 1) returns for customers would be twice that of NW Natural's cost of capital or exceeding twice that cost of capital; 2) the Company might be overstating the incremental costs of a CHP project and thus overstating the costs needing payback; and 3) customers have a benefit, due to improved power reliability, associated with building CHP which is not identified in the Company's payback computations. (*Staff/400, St. Brown/3*).

Q. How do you respond to these new issues?

A. First, regarding the returns for customers, the Company's 7.78% after tax cost of capital (or twice that much) does not represent the IRR required for corporate

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1-503-226-4211

investment in energy efficiency or CHP. As mentioned above, half of the respondents in the Pew Survey reported an IRR criteria greater than 15% for energy efficiency projects, which are viewed as low risk investments compared to CHP.

Second, the Company is not overstating the incremental costs of a CHP Project. The estimates are based on vendor supplied data. For all but the 45 megawatt (MW) prototype, WSU used the 2010 U.S. Energy Information Administration (EIA) data. The installed costs are based on EIA estimates for a packaged system cost plus hot water interconnections, grid interconnection, site labor and materials, construction management, engineering, permitting, fees, contingency, and interest during construction. The EIA data is based on a study done by Oakridge National Labs (ORNL) where they monitored the installation of 281 sites which accounted for expected variations from site to site. The breakdown of installed costs for the 45 MW prototype is based on EPA data compiled by ICF from vendor-supplied data and published in the 2014 Catalog of CHP Technologies, Technology Characterization, Combustion Turbines, March 2015.

Third, power quality is a potential benefit of CHP, however that benefit exists presently and is not improved by the proposed program. The lack of a market for natural-gas fueled CHP in Oregon suggests that the power quality benefit has not motivated customers to install CHP. In addition, Staff has not proposed any way to quantify this benefit or explained how it would impact a customer's decision to install CHP when the payback term or IRR are otherwise not acceptable.

Q. Has the Company performed any additional modeling of the IRR and payback using the updated eGRID numbers described above?

A. Yes, we have. NW Natural requested that WSU re-run its model with just released 2012 eGrid non-baseload emissions to evaluate IRR and Payback. This is attached as Exhibit NWN/504. The table below summarizes the model using the base case at current Energy Trust of Oregon incentive levels and models a $30 per MTCO2(e) incentive and $0.00 incentive. The project IRRs and Paybacks are compared for the 2010 and updated 2012 eGrid recommendations.

Even though NW Natural does not believe the EPA estimates overstated the costs of the 45 MW prototype, NW Natural requested that WSU add a 45 MW prototype at 70% of EPA reported installed costs to illustrate the impact on payback and IRR. The Company added $2 Million to the estimate for the 45 MW units and $1.2 million to the estimate for the 21.7 MW unit to account for the potential need for compression.

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1-503-226-4211

Base Case (66%)

Prototype	IRR		Payback		Zero 844 Incentive (Unaffected by eGrid)	
	2010	2012	2010	2012	IRR	Payback
Hospital - 800,000 sf with Two 800 kW Recip Engines	10.6%	12.7%	6.2	5.6	4.9%	8.9
Reciprocating Engine - 500 kW	12.2%	14.9%	5.7	5.0	5.2%	8.7
Reciprocating Engine - 4.3 MW	28.9%	32.0%	2.9	2.7	18.7%	3.9
Gas Turbine - 21.7 MW, without compression	22.3%	Not Run	4.1	Not Run	13.7%	5.4
Gas Turbine - 21.7 MW, with compression	20.9%	21.7%	4.3	4.3	12.7%	5.7
Gas Turbine - 45 MW, without compression	20.2%	Not Run	4.4	Not Run	11.9%	5.8
Gas Turbine - 45 MW, with compression	19.9%	19.9%	4.5	4.5	11.2%	6.0
Gas Turbine - 45 MW, 70% CapEx, with Compression	33.8%	35.2%	3.0	3.0	20.6%	4.0

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Technical Potential (100% - 95% Availability and 100% utilization of available waste heat)

Prototype	IRR		Payback		Zero 844 Incentive (Unaffected by eGrid)	
	2010	2012	2010	2012	IRR	Payback
Hospital - 800,000 sf with Two 800 kW Recip Engines	13.6%	16.7%	5.3	4.7	4.9%	8.9
Reciprocating Engine - 500 kW	15.9%	19.9%	4.8	4.1	5.2%	8.7
Reciprocating Engine - 4.3 MW	34.6%	39.5%	2.6	2.4	18.7%	3.9
Gas Turbine - 21.7 MW, without compression	27.2%	Not Run	3.7	Not Run	13.7%	5.4
Gas Turbine - 21.7 MW, with compression	25.5%	26.8%	3.9	3.8	12.7%	5.7
Gas Turbine - 45 MW, without compression	24.9%	Not Run	3.9	Not Run	11.9%	5.8
Gas Turbine – 45 MW, with compression	24.9%	24.9%	4.0	4.0	11.2%	6.0
Gas Turbine - 45 MW, 70% CapEx, with compression	41.6%	44.0%	2.7	2.6	20.6%	4.0

Q. Staff cites the San Diego Gas and Electric (SDGE) CHP Request for Offers (RFO) as an instance where a competitive approach was used to determine

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the appropriate level of incentives for a CHP Program to stimulate the market. (*Staff/400, St. Brown/10*). Do you agree?

A. No. The California CHP market is a mature market as a result California's Standard Offer 4 Contracts in response to the Public Utility Regulatory Policies Act (PURPA). There are 4,994 MWs of operating natural gas fueled qualifying facility (QF) Status Cogeneration facilities today in California. The San Diego Gas and Electric RFO is designed to transition from the existing QF CHP PURPA Program for larger CHP units. The Transition Period is a period in which a CHP Facility will either obtain a new power purchase agreement, sell into the wholesale market, shut down, or cease to export to the grid. The Oregon cogeneration market is nothing like the California cogeneration market. NW Natural is attempting to stimulate the Oregon CHP market to achieve a reduction in Carbon emissions. Staff's comparison is overly broad and is not analogous to the current state of CHP in Oregon.

Q. Staff continues to advocate for a reverse auction for the CHP Program. Why does the Company believe that a reverse auction would impair the CHP Program

A. In addition to the reasons provided in the Application and earlier rounds of testimony, reverse auctions will not be effective for the CHP Program because reverse auctions are most effective in highly competitive markets when the requirements are simple. In a June 1, 2015, Memorandum to Chief Officers and

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1-503-226-4211

Senior Procurement Executives, from Anne Rung, Administrator, Executive Office of the President, Office of Management and Budget regarding the Effective Use of Reverse Auctions, Executive Director Rung makes this point:

> <u>Is the requirement suited for a reverse auction?</u> Reverse auctions are not a one-size-fits- all tool. Reverse auctions are likely to be most effective in a highly competitive marketplace when requirements are steady and relatively simple and might otherwise be acquired using either a sealed bid or achieving best value through "low price technically acceptable" source selection criteria, and result in fixed price agreements. These circumstances would typically exist in acquisitions for commercial items and simple services that often fall under the [simplified acquisition threshold]. As with any procurement, market research must be conducted to understand the marketplace and to determine if it is reasonable to assume that the potential benefits of a reverse auction can be achieved. (*NWN/505, Summers/*2).

In the case of carbon reduction, a competitive market does not exist and no regulatory mandates or laws exist that require commercial or industrial industries to reduce carbon emissions at the state or federal levels.

Additionally, the reverse auction concept is designed to drive prices down to a single award. A single award is not consistent with the objectives of the proposed program. NW Natural's program is designed to award multiple customers with the goal of providing certainty and technical support to encourage broad interest and participation, until such point where NW Natural has reached its base case.

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V. RISK SHARING

Q. Staff has proposed that NW Natural should share in the cost risks associated with the CHP Program if the program is poorly administered or mismanaged. (*Staff/300, Klotz/4*). Do you believe this request is appropriate, specifically for the CHP Program?

A. No. The costs of the CHP Program are tied to the success of the program. NW Natural will only pay participants for measured and verified carbon savings from CHP. If the program is not successful – for whatever reason – our ratepayers will only pay a proportional amount in relation to the carbon savings. For instance, if the Company only reaches 50% of the base case carbon reductions, customers will only pay for those savings.

VI. CONCLUSION

Q. Does this conclude your testimony?

A. Yes.

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UM 1744
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Combined Heat & Power (CHP)

eGRID 2012
Summary Tables

October 16, 2015

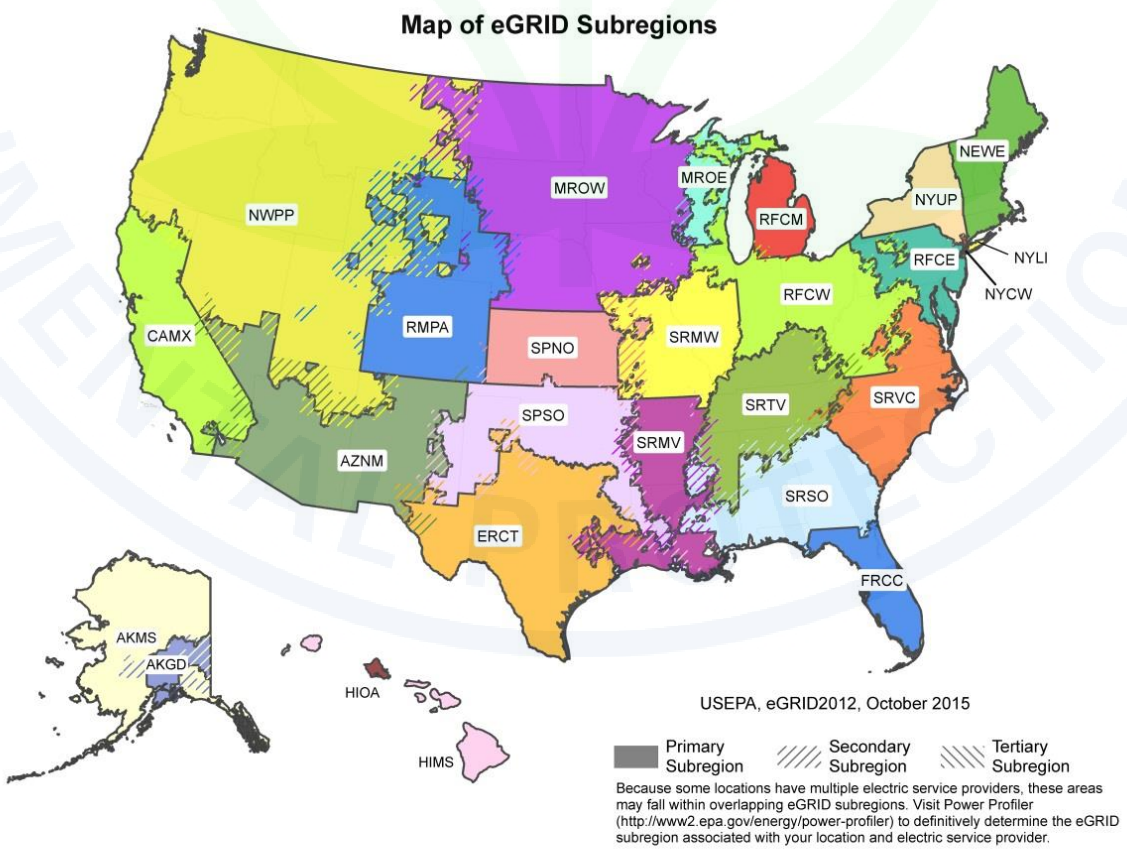
eGRID2012
Summary Tables

(created 10/05/15)

1. eGRID2012 Subregion Emissions - Greenhouse Gases
2. eGRID2012 Subregion Emissions - Criteria Pollutants
3. eGRID2012 Subregion Output Emission Rates - Greenhouse Gases
4. eGRID2012 Subregion Output Emission Rates - Criteria Pollutants
5. eGRID2012 Subregion Resource Mix
6. eGRID2012 NERC Region Emissions
7. eGRID2012 NERC Region Output Emission Rates
8. eGRID2012 NERC Region Resource Mix
9. eGRID2012 Grid Gross Loss (%)
10. eGRID2012 State Emissions and Input Emission Rates
11. eGRID2012 State Resource Mix
12. eGRID2012 Generation by Fuel Type and CO_2 Emission Rates

1. eGRID2012 Subregion Emissions – Greenhouse Gases

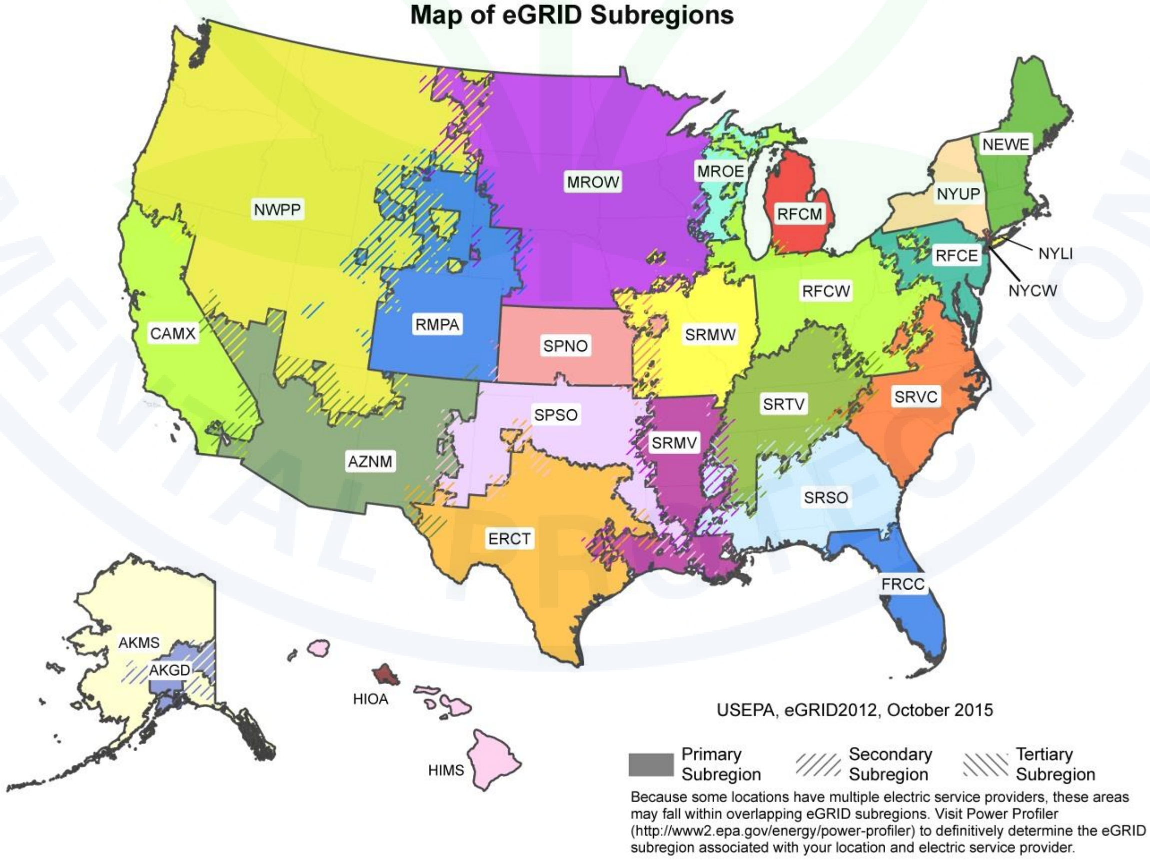
eGRID subregion acronym	eGRID subregion name	Carbon dioxide (CO_2)		Methane (CH_4)		Nitrous oxide (N_2O)		Carbon dioxide equivalent (CO_2e)	
		Emissions (tons)	Total output emission rate (lb/MWh)	Emissions (lbs)	Total output emission rate (lb/GWh)	Emissions (lbs)	Total output emission rate (lb/GWh)	Emissions (tons)	Total output emission rate (lb/MWh)
AKGD	ASCC Alaska Grid	3,382,037.0	1,268.73	140,402.7	26.34	40,490.5	7.59	3,389,787.2	1,271.64
AKMS	ASCC Miscellaneous	384,195.8	481.17	29,787.0	18.65	5,666.3	3.55	385,386.8	482.66
AZNM	WECC Southwest	102,534,225.3	1,152.89	3,317,864.6	18.65	2,686,986.1	15.11	102,985,545.7	1,157.96
CAMX	WECC California	67,187,988.1	650.31	6,429,630.8	31.12	1,172,434.9	5.67	67,437,084.4	652.72
ERCT	ERCOT All	205,873,315.5	1,143.04	6,015,952.8	16.70	4,443,235.0	12.33	206,625,056.6	1,147.21
FRCC	FRCC All	118,861,947.3	1,125.35	8,459,346.4	40.05	2,503,826.1	11.85	119,338,507.3	1,129.86
HIMS	HICC Miscellaneous	1,760,031.8	1,200.10	199,673.8	68.08	37,202.0	12.68	1,767,894.6	1,205.46
HIOA	HICC Oahu	5,939,881.8	1,576.38	681,311.9	90.41	162,405.3	21.55	5,972,208.4	1,584.95
MROE	MRO East	21,794,875.8	1,522.57	695,782.7	24.30	731,606.9	25.55	21,915,580.6	1,531.00
MROW	MRO West	145,305,369.2	1,425.15	5,627,262.8	27.60	4,947,215.7	24.26	146,130,871.2	1,433.25
NEWE	NPCC New England	38,377,520.5	637.90	8,764,225.4	72.84	1,288,397.3	10.71	38,669,246.4	642.75
NWPP	WECC Northwest	95,734,309.7	665.75	3,622,959.4	12.60	2,983,818.8	10.38	96,234,699.4	669.23
NYCW	NPCC NYC/Westchester	15,851,201.7	696.70	1,160,747.0	25.51	133,430.3	2.93	15,882,764.1	698.08
NYLI	NPCC Long Island	7,280,232.8	1,201.20	947,931.1	78.20	119,618.7	9.87	7,308,726.9	1,205.90
NYUP	NPCC Upstate NY	16,873,346.4	408.80	1,287,300.2	15.59	315,913.7	3.83	16,935,829.7	410.31
RFCE	RFC East	112,888,707.9	858.56	6,954,055.7	26.44	3,020,840.1	11.49	113,429,807.1	862.68
RFCM	RFC Michigan	68,119,780.7	1,569.23	2,635,889.2	30.36	2,093,696.0	24.12	68,471,962.7	1,577.34
RFCW	RFC West	391,126,291.4	1,379.48	9,701,816.8	17.11	12,286,300.3	21.67	393,132,519.0	1,386.55
RMPA	WECC Rockies	57,993,856.1	1,822.65	1,378,226.1	21.66	1,790,072.3	28.13	58,285,775.9	1,831.82
SPNO	SPP North	59,782,627.7	1,721.65	1,403,934.9	20.22	1,885,096.3	27.14	60,089,349.8	1,730.49
SPSO	SPP South	117,500,299.0	1,538.63	3,627,540.2	23.75	3,050,862.7	19.98	118,011,271.9	1,545.32
SRMV	SERC Mississippi Valley	95,886,176.4	1,052.92	3,816,210.1	20.95	1,931,912.9	10.61	96,225,693.1	1,056.65
SRMW	SERC Midwest	113,709,694.8	1,710.75	2,603,196.3	19.58	3,655,614.1	27.50	114,303,633.0	1,719.68
SRSO	SERC South	146,477,427.2	1,149.05	5,777,614.3	22.66	3,948,687.2	15.49	147,150,138.6	1,154.32
SRTV	SERC Tennessee Valley	153,167,116.4	1,337.15	3,982,959.3	17.39	4,761,521.4	20.78	153,946,973.3	1,343.96
SRVC	SERC Virginia/Carolina	135,132,027.1	932.87	6,937,947.2	23.95	4,229,617.5	14.60	135,860,466.3	937.90
U.S.		**2,298,924,483.4**	**1,136.53**	**96,199,568.7**	**23.78**	**64,226,468.3**	**15.88**	**2,309,886,780.4**	**1,141.95**



2. eGRID2012 Subregion Emissions – Criteria Pollutants

eGRID subregion acronym	eGRID subregion name	Nitrogen oxides (NO_x)				Sulfur dioxide (SO_2)	
		Emissions (tons)	Total output emission rate (lb/MWh)	Ozone season emissions (tons)	Ozone season total output emission rate (lb/MWh)	Emissions (tons)	Total output emission rate (lb/MWh)
AKGD	ASCC Alaska Grid	6,915.85	2.5944	2,806.53	2.8050	2,108.78	0.7911
AKMS	ASCC Miscellaneous	4,812.53	6.0272	2,012.32	6.9876	275.47	0.3450
AZNM	WECC Southwest	115,206.61	1.2954	52,010.92	1.2331	39,065.16	0.4392
CAMX	WECC California	34,618.62	0.3351	15,715.99	0.3307	20,665.19	0.2000
ERCT	ERCOT All	109,604.19	0.6085	52,291.10	0.5994	346,399.70	1.9233
FRCC	FRCC All	65,764.23	0.6226	32,261.83	0.6497	134,754.30	1.2758
HIMS	HICC Miscellaneous	7,464.00	5.0894	2,947.14	4.8250	5,732.57	3.9088
HIOA	HICC Oahu	7,869.41	2.0885	3,309.46	2.0767	19,550.87	5.1886
MROE	MRO East	17,598.10	1.2294	8,484.76	1.2619	59,760.10	4.1748
MROW	MRO West	164,050.95	1.6090	69,021.42	1.5795	299,484.96	2.9373
NEWE	NPCC New England	24,559.21	0.4082	8,725.84	0.3221	60,433.63	1.0045
NWPP	WECC Northwest	104,109.15	0.7240	41,249.74	0.6687	109,096.30	0.7587
NYCW	NPCC NYC/Westchester	7,583.66	0.3333	3,652.65	0.3396	1,458.43	0.0641
NYLI	NPCC Long Island	4,376.57	0.7221	2,479.14	0.7690	5,949.92	0.9817
NYUP	NPCC Upstate NY	11,393.39	0.2760	5,027.66	0.2818	26,821.30	0.6498
RFCE	RFC East	104,919.63	0.7980	50,809.70	0.8539	185,487.76	1.4107
RFCM	RFC Michigan	65,732.87	1.5142	29,263.66	1.4687	196,167.89	4.5190
RFCW	RFC West	341,864.18	1.2057	152,302.40	1.2266	961,849.06	3.3924
RMPA	WECC Rockies	62,952.61	1.9785	27,836.57	2.0068	51,254.96	1.6109
SPNO	SPP North	47,993.46	1.3821	23,430.37	1.3908	59,998.41	1.7279
SPSO	SPP South	125,199.34	1.6394	59,061.00	1.6322	194,323.79	2.5446
SRMV	SERC Mississippi Valley	89,229.17	0.9798	43,888.72	1.0275	134,574.22	1.4777
SRMW	SERC Midwest	85,901.12	1.2924	40,059.74	1.3795	212,369.32	3.1951
SRSO	SERC South	96,692.66	0.7585	45,953.42	0.7621	274,933.11	2.1567
SRTV	SERC Tennessee Valley	110,837.27	0.9676	51,044.65	0.9966	259,061.01	2.2616
SRVC	SERC Virginia/Carolina	96,438.08	0.6658	46,098.34	0.6916	155,846.09	1.0759
U.S.		**1,913,686.86**	**0.9461**	**871,745.09**	**0.9460**	**3,817,422.30**	**1.8872**

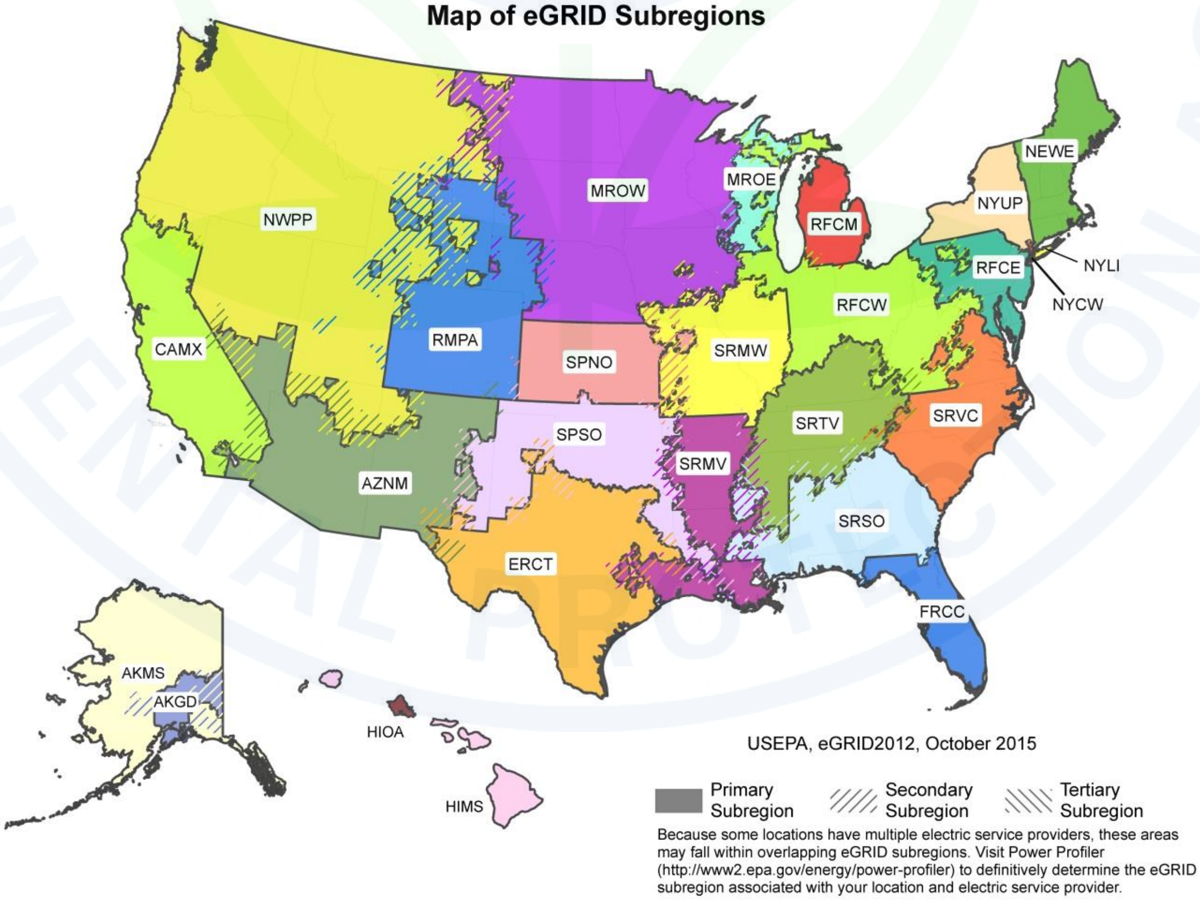
Map of eGRID Subregions



3. eGRID2012 Subregion Output Emission Rates – Greenhouse Gases

eGRID subregion acronym	eGRID subregion name	Total output emission rates			Fossil fuel output emission rate	Non-baseload output emission rates		
		CO_2 (lb/MWh)	CH_4 (lb/GWh)	N_2O (lb/GWh)	CO_2 (lb/MWh)	CO_2 (lb/MWh)	CH_4 (lb/GWh)	N_2O (lb/GWh)
AKGD	ASCC Alaska Grid	1,268.73	26.34	7.59	1,413.52	1,377.77	28.66	3.38
AKMS	ASCC Miscellaneous	481.17	18.65	3.55	1,400.38	1,404.49	55.64	10.70
AZNM	WECC Southwest	1,152.89	18.65	15.11	1,613.86	1,236.02	21.56	10.52
CAMX	WECC California	650.31	31.12	5.67	986.41	1,018.87	37.61	6.04
ERCT	ERCOT All	1,143.04	16.70	12.33	1,418.13	1,280.59	21.53	10.71
FRCC	FRCC All	1,125.35	40.05	11.85	1,216.71	1,333.93	38.81	13.79
HIMS	HICC Miscellaneous	1,200.10	68.08	12.68	1,656.12	1,331.47	96.82	17.15
HIOA	HICC Oahu	1,576.38	90.41	21.55	1,582.88	1,402.27	118.01	19.43
MROE	MRO East	1,522.57	24.30	25.55	2,077.12	1,739.00	30.17	26.26
MROW	MRO West	1,425.15	27.60	24.26	2,152.46	1,965.21	52.60	32.72
NEWE	NPCC New England	637.90	72.84	10.71	980.27	1,079.73	67.70	12.90
NWPP	WECC Northwest	665.75	12.60	10.38	1,858.75	1,579.07	38.30	22.84
NYCW	NPCC NYC/Westchester	696.70	25.51	2.93	1,175.61	1,081.11	22.50	2.32
NYLI	NPCC Long Island	1,201.20	78.20	9.87	1,129.27	1,303.42	31.40	3.56
NYUP	NPCC Upstate NY	408.80	15.59	3.83	1,085.63	1,228.56	39.00	13.04
RFCE	RFC East	858.56	26.44	11.49	1,469.42	1,492.01	32.74	18.69
RFCM	RFC Michigan	1,569.23	30.36	24.12	1,853.55	1,856.21	33.91	28.72
RFCW	RFC West	1,379.48	17.11	21.67	1,942.40	1,791.71	21.76	27.85
RMPA	WECC Rockies	1,822.65	21.66	28.13	2,094.71	1,669.58	22.89	20.66
SPNO	SPP North	1,721.65	20.22	27.14	2,149.67	2,112.08	26.11	30.63
SPSO	SPP South	1,538.63	23.75	19.98	1,729.36	1,590.13	27.60	16.19
SRMV	SERC Mississippi Valley	1,052.92	20.95	10.61	1,384.45	1,301.65	27.43	9.75
SRMW	SERC Midwest	1,710.75	19.58	27.50	2,069.72	1,917.96	23.29	28.84
SRSO	SERC South	1,149.05	22.66	15.49	1,518.99	1,696.79	28.17	24.83
SRTV	SERC Tennessee Valley	1,337.15	17.39	20.78	1,912.59	1,743.96	22.84	26.11
SRVC	SERC Virginia/Carolina	932.87	23.95	14.60	1,665.71	1,790.57	53.10	29.94
U.S.		**1,136.53**	**23.78**	**15.88**	**1,640.13**	**1,549.36**	**30.99**	**19.86**

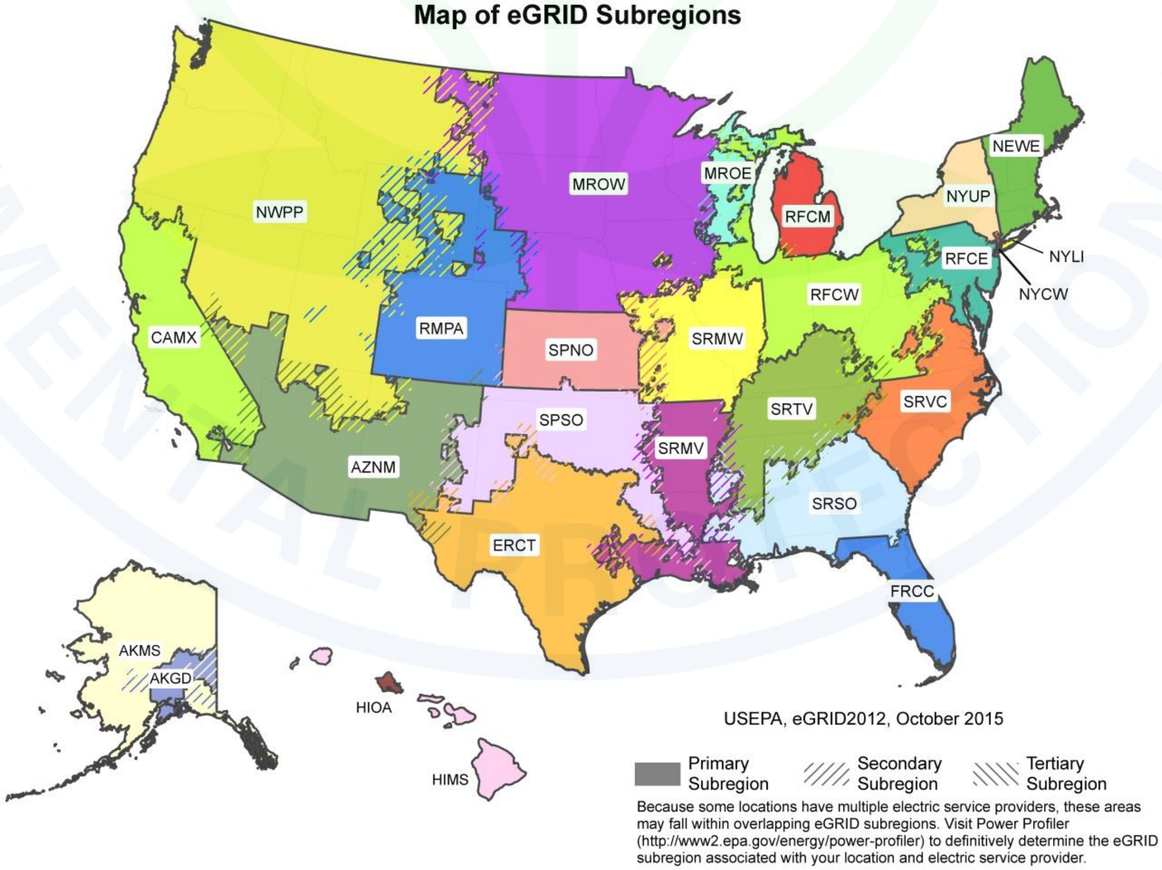
Map of eGRID Subregions



4. eGRID2012 Subregion Output Emission Rates – Criteria Pollutants

eGRID subregion acronym	eGRID subregion name	Total output emission rates			Fossil fuel output emission rates			Non-baseload output emission rates		
		NO_x (lb/MWh)	Ozone season NO_x (lb/MWh)	SO_2 (lb/MWh)	NO_x (lb/MWh)	Ozone season NO_x (lb/MWh)	SO_2 (lb/MWh)	NO_x (lb/MWh)	Ozone season NO_x (lb/MWh)	SO_2 (lb/MWh)
AKGD	ASCC Alaska Grid	2.5944	2.8050	0.7911	2.8905	3.1031	0.8814	2.5108	2.2915	0.3088
AKMS	ASCC Miscellaneous	6.0272	6.9876	0.3450	17.5415	17.8896	1.0041	18.6055	18.7183	1.0812
AZNM	WECC Southwest	1.2954	1.2331	0.4392	1.8060	1.6804	0.6052	1.0309	1.1602	0.3253
CAMX	WECC California	0.3351	0.3307	0.2000	0.4621	0.4732	0.1901	0.3514	0.4355	0.2790
ERCT	ERCOT All	0.6085	0.5994	1.9233	0.7540	0.7226	2.3855	0.7249	0.8523	2.0299
FRCC	FRCC All	0.6226	0.6497	1.2758	0.5979	0.6319	0.8298	0.9167	1.1777	1.6026
HIMS	HICC Miscellaneous	5.0894	4.8250	3.9088	6.9669	6.9676	5.3730	3.1897	3.0464	4.7750
HIOA	HICC Oahu	2.0885	2.0767	5.1886	2.0408	2.0182	5.3114	2.3760	2.4102	3.7587
MROE	MRO East	1.2294	1.2619	4.1748	1.5928	1.5747	5.5208	1.6954	1.8106	4.7298
MROW	MRO West	1.6090	1.5795	2.9373	2.3633	2.3016	4.4187	2.5376	2.3880	4.9129
NEWE	NPCC New England	0.4082	0.3221	1.0045	0.2805	0.2506	0.4033	0.6140	0.6182	1.3580
NWPP	WECC Northwest	0.7240	0.6687	0.7587	1.9805	2.0195	1.9581	1.5959	1.5030	1.6177
NYCW	NPCC NYC/Westchester	0.3333	0.3396	0.0641	0.4625	0.4622	0.0174	0.6319	0.8141	0.0270
NYLI	NPCC Long Island	0.7221	0.7690	0.9817	0.6069	0.6826	0.1798	0.8688	1.2600	0.3689
NYUP	NPCC Upstate NY	0.2760	0.2818	0.6498	0.6121	0.5658	1.2271	1.0062	1.3064	2.3801
RFCE	RFC East	0.7980	0.8539	1.4107	1.3537	1.3676	1.9385	1.4677	1.8031	2.5083
RFCM	RFC Michigan	1.5142	1.4687	4.5190	1.7460	1.6503	5.2944	1.8566	1.9122	5.6384
RFCW	RFC West	1.2057	1.2266	3.3924	1.6892	1.6721	4.7439	1.6493	1.7673	5.7097
RMPA	WECC Rockies	1.9785	2.0068	1.6109	2.2733	2.2827	1.8509	2.2328	2.5370	1.5147
SPNO	SPP North	1.3821	1.3908	1.7279	1.7148	1.7058	2.1437	2.0950	2.4636	3.0002
SPSO	SPP South	1.6394	1.6322	2.5446	1.8243	1.7605	2.8168	1.8101	2.2179	1.8000
SRMV	SERC Mississippi Valley	0.9798	1.0275	1.4777	1.2474	1.2715	1.7843	1.3977	1.7023	1.2369
SRMW	SERC Midwest	1.2924	1.3795	3.1951	1.5634	1.6381	3.8652	1.2956	1.3816	3.3130
SRSO	SERC South	0.7585	0.7621	2.1567	0.9410	0.9139	2.7071	1.4589	1.8026	4.6878
SRTV	SERC Tennessee Valley	0.9676	0.9966	2.2616	1.3733	1.3649	3.1953	1.2932	1.5146	2.9791
SRVC	SERC Virginia/Carolina	0.6658	0.6916	1.0759	1.1051	1.1170	1.7143	1.6038	1.8497	3.3203
U.S.		**0.9461**	**0.9460**	**1.8872**	**1.3268**	**1.2959**	**2.5741**	**1.3555**	**1.5557**	**2.9317**

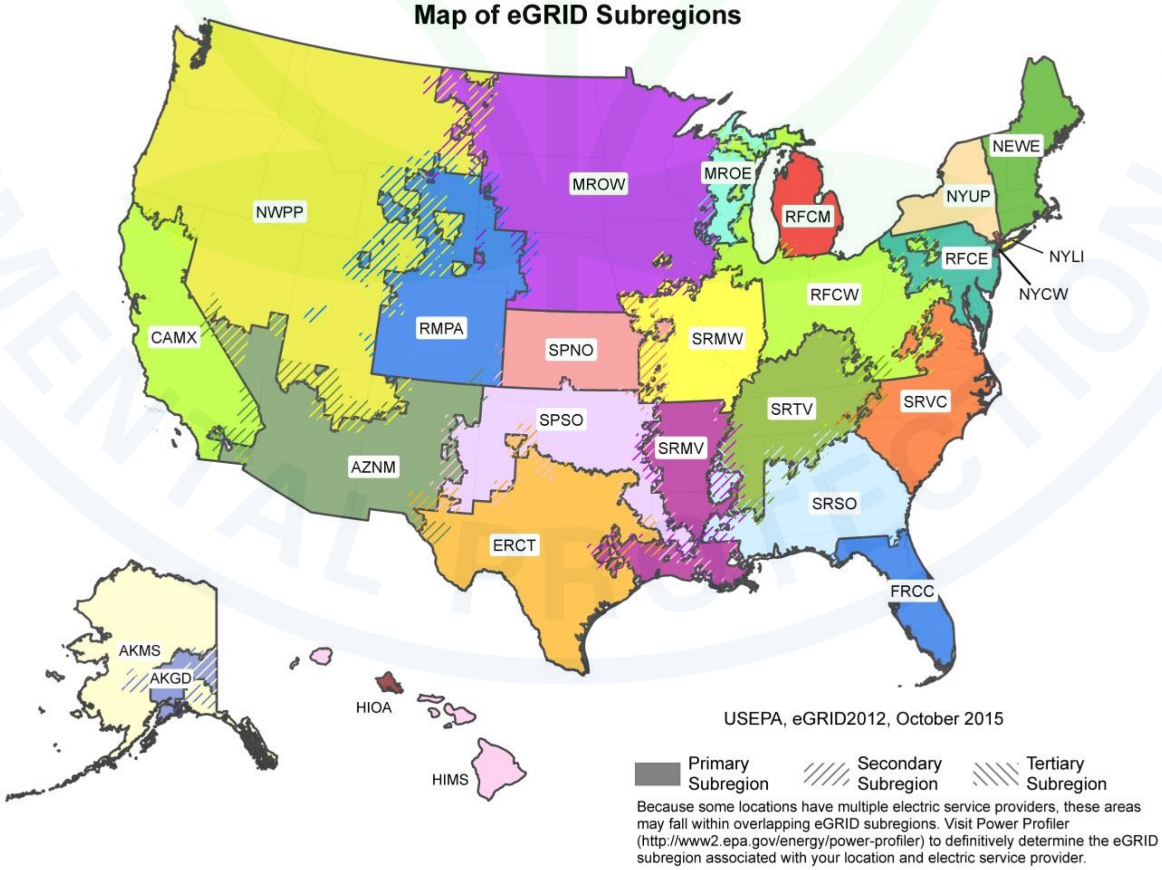
Map of eGRID Subregions



5. eGRID2012 Subregion Resource Mix

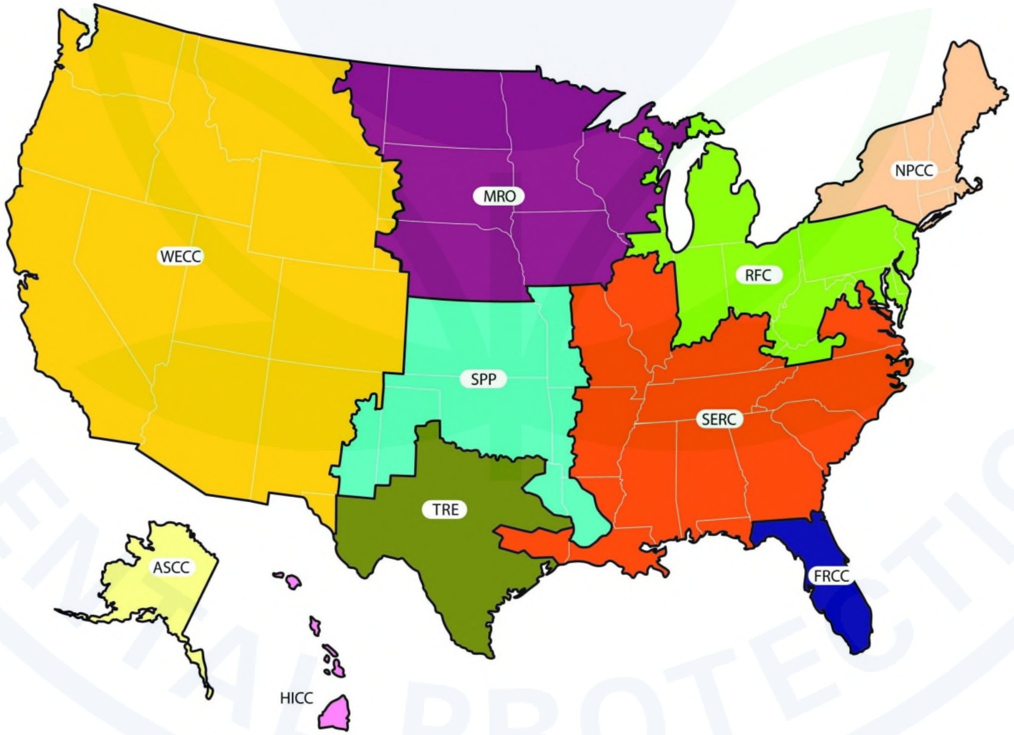
eGRID subregion acronym	eGRID subregion name	Nameplate capacity (MW)	Net generation (MWh)	Generation Resource Mix (percent)										
				Coal	Oil	Gas	Other fossil	Nuclear	Hydro	Biomass	Wind	Solar	Geo-thermal	Other unknown/ purchased fuel
AKGD	ASCC Alaska Grid	2,007.8	5,331,368.0	12.8477	11.5119	65.3975	0.0000	0.0000	10.2429	0.0000	0.0000	0.0000	0.0000	0.0000
AKMS	ASCC Miscellaneous	754.2	1,596,926.5	0.0000	26.5523	7.6469	0.0000	0.0000	64.4336	0.1606	1.2066	0.0000	0.0000	0.0000
AZNM	WECC Southwest	63,160.5	177,873,710.9	37.3633	0.0501	33.9397	0.0042	17.9531	6.3295	0.3291	0.9724	0.6563	2.3956	0.0067
CAMX	WECC California	95,000.9	206,633,044.0	5.3301	0.8232	58.5863	0.0875	8.9567	12.7375	2.8533	5.0012	0.8732	4.4331	0.3180
ERCT	ERCOT All	115,223.9	360,221,517.3	30.5073	0.9452	49.0477	0.1204	10.6715	0.1091	0.1977	8.2871	0.0328	0.0000	0.0812
FRCC	FRCC All	78,701.1	211,244,527.5	19.4235	0.6443	68.0575	0.6566	8.4594	0.0712	1.7642	0.0000	0.0917	0.0000	0.8317
HIMS	HICC Miscellaneous	974.2	2,933,143.4	1.3576	64.2117	0.0000	7.3575	0.0000	3.9064	3.6304	10.4875	0.1507	8.8982	0.0000
HIOA	HICC Oahu	2,107.4	7,536,125.3	19.8712	74.9241	0.0000	1.8820	0.0000	0.0000	2.3830	0.9371	0.0025	0.0000	0.0000
MROE	MRO East	10,323.2	28,629,056.0	64.3153	0.9998	7.8554	0.1644	15.7738	2.9180	3.7800	4.0806	0.0000	0.0000	0.1126
MROW	MRO West	61,555.1	203,915,893.0	60.8336	0.1281	5.0019	0.1446	10.8341	6.2900	1.2954	15.2138	0.0000	0.0000	0.2584
NEWE	NPCC New England	40,761.9	120,324,524.1	2.9468	0.3392	51.9358	1.6642	30.0154	5.8701	6.0580	1.0680	0.0275	0.0000	0.0748
NWPP	WECC Northwest	80,235.0	287,596,498.3	24.5037	0.3463	10.6587	0.1333	3.2454	52.2177	1.0982	7.0260	0.0040	0.6476	0.1192
NYCW	NPCC NYC/Westchester	14,988.5	45,503,844.6	0.0000	0.1812	61.6948	0.4255	37.2211	0.0032	0.4741	0.0000	0.0000	0.0000	0.0000
NYLI	NPCC Long Island	6,031.2	12,121,635.9	0.0000	2.8882	89.2010	3.5290	0.0000	0.0000	3.9470	0.0000	0.4349	0.0000	0.0000
NYUP	NPCC Upstate NY	28,527.0	82,550,860.0	5.5130	0.1820	30.3999	0.3818	28.8761	29.2443	1.7995	3.6034	0.0000	0.0000	0.0000
RFCE	RFC East	81,434.8	262,972,203.0	23.8506	0.4047	30.7631	0.6749	40.9183	1.1175	1.3829	0.7618	0.1262	0.0000	0.0000
RFCM	RFC Michigan	30,753.9	86,819,386.1	58.5744	0.3601	24.9262	0.7525	11.8643	-0.3321	2.0364	1.8182	0.0000	0.0000	0.0000
RFCW	RFC West	165,405.0	567,064,674.2	58.7362	0.5280	11.0509	0.6630	25.7250	0.6682	0.5006	2.0570	0.0136	0.0000	0.0575
RMPA	WECC Rockies	19,921.2	63,636,839.6	70.3646	0.0411	16.6244	0.0000	0.0000	3.1724	0.0911	9.3627	0.2567	0.0000	0.0870
SPNO	SPP North	23,788.5	69,447,958.9	70.6814	0.0918	9.8012	0.0285	11.9297	0.0981	0.0873	7.2821	0.0000	0.0000	0.0000
SPSO	SPP South	50,658.9	152,734,002.2	48.4033	0.7668	39.4001	0.1997	0.0000	2.0027	1.4982	7.6329	0.0770	0.0000	0.0193
SRMV	SERC Mississippi Valley	52,017.2	182,134,134.3	20.5889	1.2729	53.5965	0.7162	21.1099	0.8429	1.7362	0.0000	0.0000	0.0000	0.1366
SRMW	SERC Midwest	38,922.6	132,935,700.9	75.4034	0.0680	6.8652	0.1280	15.1141	0.2213	0.0972	1.9076	0.0000	0.0000	0.1952
SRSO	SERC South	78,562.6	254,954,509.9	33.8126	0.1918	41.9257	0.0903	19.1033	1.7819	3.0938	0.0000	0.0006	0.0000	0.0000
SRTV	SERC Tennessee Valley	67,967.3	229,094,795.2	53.6644	0.7361	15.5289	0.0097	22.3402	6.9009	0.7985	0.0207	0.0006	0.0000	0.0000
SRVC	SERC Virginia/Carolina	88,528.9	289,711,035.7	34.7513	0.2012	20.2079	0.2173	41.1632	0.8794	2.4344	0.0000	0.0380	0.0000	0.1074
U.S.		**1,309,394.6**	**4,045,517,914.7**	**37.4156**	**0.7034**	**30.2949**	**0.3683**	**19.0169**	**6.7030**	**1.4404**	**3.4476**	**0.1035**	**0.3842**	**0.1221**

Map of eGRID Subregions



6. eGRID2012 NERC Region Emissions

NERC region acronym	NERC region name	Nitrogen oxides (NO_x) Emissions (tons)	Nitrogen oxides (NO_x) Total output emission rate (lb/MWh)	Nitrogen oxides (NO_x) Ozone season emissions (tons)	Nitrogen oxides (NO_x) Ozone season total output emission rate (lb/MWh)	Sulfur dioxide (SO_2) Emissions (tons)	Sulfur dioxide (SO_2) Total output emission rate (lb/MWh)	Carbon dioxide (CO_2) Emissions (tons)	Carbon dioxide (CO_2) Total output emission rate (lb/MWh)	Methane (CH_4) Emissions (lbs)	Methane (CH_4) Total output emission rate (lb/GWh)	Nitrous oxide (N_2O) Emissions (lbs)	Nitrous oxide (N_2O) Total output emission rate (lb/GWh)
ASCC	Alaska Systems Coordinating Council	11,728.39	3.3856	4,818.86	3.7399	2,384.25	0.6883	3,766,232.7	1,087.20	170,189.7	24.56	46,156.7	6.66
FRCC	Florida Reliability Coordinating Council	65,764.23	0.6226	32,261.83	0.6497	134,754.30	1.2758	118,861,947.3	1,125.35	8,459,346.4	40.05	2,503,826.1	11.85
HICC	Hawaiian Islands Coordinating Council	15,333.42	2.9292	6,256.60	2.8382	25,283.44	4.8300	7,699,913.5	1,470.96	880,985.7	84.15	199,607.3	19.07
MRO	Midwest Reliability Organization	181,649.04	1.5623	77,506.18	1.5371	359,245.06	3.0897	167,100,245.0	1,437.14	6,323,045.5	27.19	5,678,822.6	24.42
NPCC	Northeast Power Coordinating Council	47,912.82	0.3679	19,885.29	0.3376	94,663.28	0.7268	78,382,301.3	601.78	12,160,203.7	46.68	1,857,360.1	7.13
RFC	Reliability First Corporation	512,516.67	1.1180	232,375.76	1.1414	1,343,504.72	2.9307	572,134,780.0	1,248.04	19,291,761.7	21.04	17,400,836.4	18.98
SERC	SERC Reliability Corporation	479,098.31	0.8800	227,044.88	0.9084	1,036,783.74	1.9044	644,372,442.0	1,183.61	23,117,927.2	21.23	18,527,352.9	17.02
SPP	Southwest Power Pool	173,192.80	1.5590	82,491.37	1.5555	254,322.20	2.2893	177,282,926.7	1,595.84	5,031,475.1	22.65	4,935,959.0	22.22
TRE	Texas Regional Entity	109,604.19	0.6085	52,291.10	0.5994	346,399.70	1.9233	205,873,315.5	1,143.04	6,015,952.8	16.70	4,443,235.0	12.33
WECC	Western Electricity Coordinating Council	316,886.98	0.8614	136,813.23	0.8279	220,081.60	0.5983	323,450,379.3	879.25	14,748,681.0	20.05	8,633,312.1	11.73
U.S.		**1,913,686.86**	**0.9461**	**871,745.09**	**0.9460**	**3,817,422.30**	**1.8872**	**2,298,924,483.4**	**1,136.53**	**96,199,568.7**	**23.78**	**64,226,468.3**	**15.88**

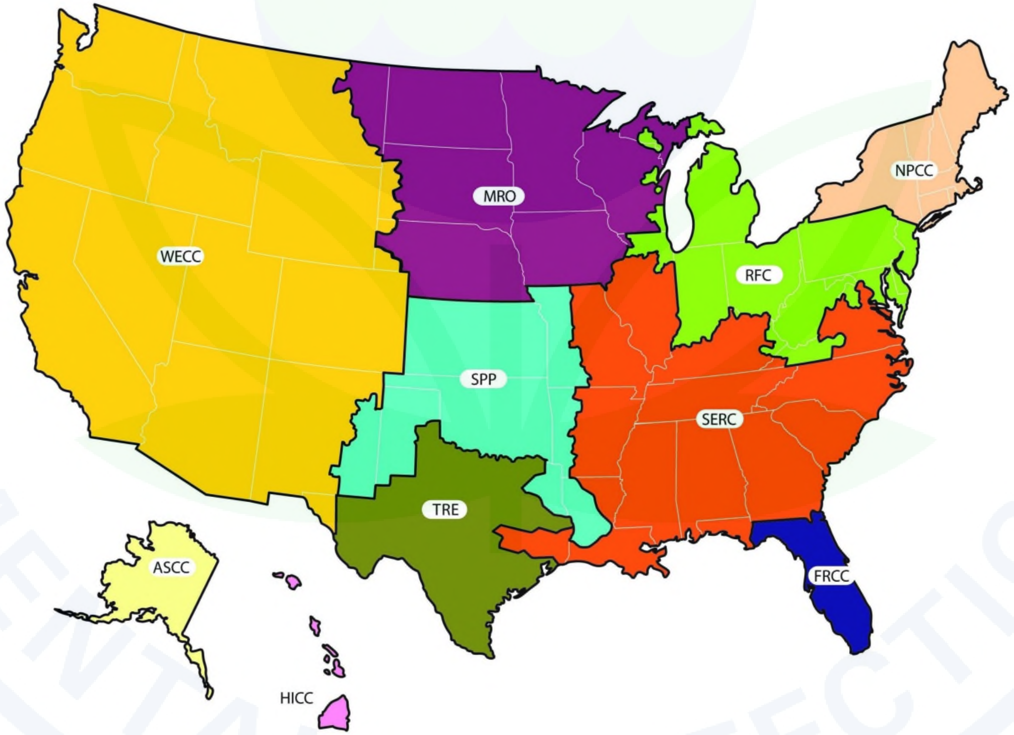


This is a representational map; many of the boundaries shown on this map are approximate because they are based on companies, not on strictly geographical boundaries.

USEPA eGRID2012

7. eGRID2012 NERC Region Output Emission Rates

NERC region acronym	NERC region name	Total output emissions rates						Fossil fuel output emission rates				Non-baseload output emission rates					
		NO_x (lb/MWh)	Ozone season NO_x (lb/MWh)	SO_2 (lb/MWh)	CO_2 (lb/MWh)	CH_4 (lb/GWh)	N_2O (lb/GWh)	NO_x (lb/MWh)	Ozone season NO_x (lb/MWh)	SO_2 (lb/MWh)	CO_2 (lb/MWh)	NO_x (lb/MWh)	Ozone season NO_x (lb/MWh)	SO_2 (lb/MWh)	CO_2 (lb/MWh)	CH_4 (lb/GWh)	N_2O (lb/GWh)
ASCC	Alaska Systems Coordinating Council	3.3856	3.7399	0.6883	1,087.20	24.56	6.66	4.3976	4.7388	0.8940	1,412.17	8.4473	7.0044	0.59	1,387.62	38.61	6.08
FRCC	Florida Reliability Coordinating Council	0.6226	0.6497	1.2758	1,125.35	40.05	11.85	0.5979	0.6319	0.8298	1,216.71	0.9167	1.1777	1.60	1,333.93	38.81	13.79
HICC	Hawaiian Islands Coordinating Council	2.9292	2.8382	4.8300	1,470.96	84.15	19.07	3.1582	3.0832	5.3254	1,599.49	2.5679	2.5622	4.00	1,385.57	113.01	18.89
MRO	Midwest Reliability Organization	1.5623	1.5371	3.0897	1,437.14	27.19	24.42	2.2599	2.1934	4.5666	2,142.35	2.3966	2.2883	4.88	1,927.33	48.85	31.64
NPCC	Northeast Power Coordinating Council	0.3679	0.3376	0.7268	601.78	46.68	7.13	0.4175	0.4036	0.4849	1,056.01	0.7448	0.8939	1.28	1,141.31	48.32	10.05
RFC	Reliability First Corporation	1.1180	1.1414	2.9307	1,248.04	21.04	18.98	1.6174	1.5947	4.1529	1,821.32	1.6287	1.7932	4.91	1,724.93	25.92	25.68
SERC	SERC Reliability Corporation	0.8800	0.9084	1.9044	1,183.61	21.23	17.02	1.2125	1.2141	2.6013	1,688.12	1.4155	1.6727	3.16	1,670.65	30.92	23.50
SPP	Southwest Power Pool	1.5590	1.5555	2.2893	1,595.84	22.65	22.22	1.7923	1.7445	2.6202	1,852.14	1.8751	2.2775	2.07	1,709.23	27.26	19.48
TRE	Texas Regional Entity	0.6085	0.5994	1.9233	1,143.04	16.70	12.33	0.7540	0.7226	2.3855	1,418.13	0.7249	0.8523	2.03	1,280.59	21.53	10.71
WECC	Western Electricity Coordinating Council	0.8614	0.8279	0.5983	879.25	20.05	11.73	1.4806	1.4442	0.9681	1,536.66	1.0338	1.1242	0.71	1,278.28	31.09	12.69
U.S.		**0.9461**	**0.9460**	**1.8872**	**1,136.53**	**23.78**	**15.88**	**1.3268**	**1.2959**	**2.5741**	**1,640.13**	**1.3555**	**1.5557**	**2.9317**	**1,549.36**	**30.99**	**19.86**



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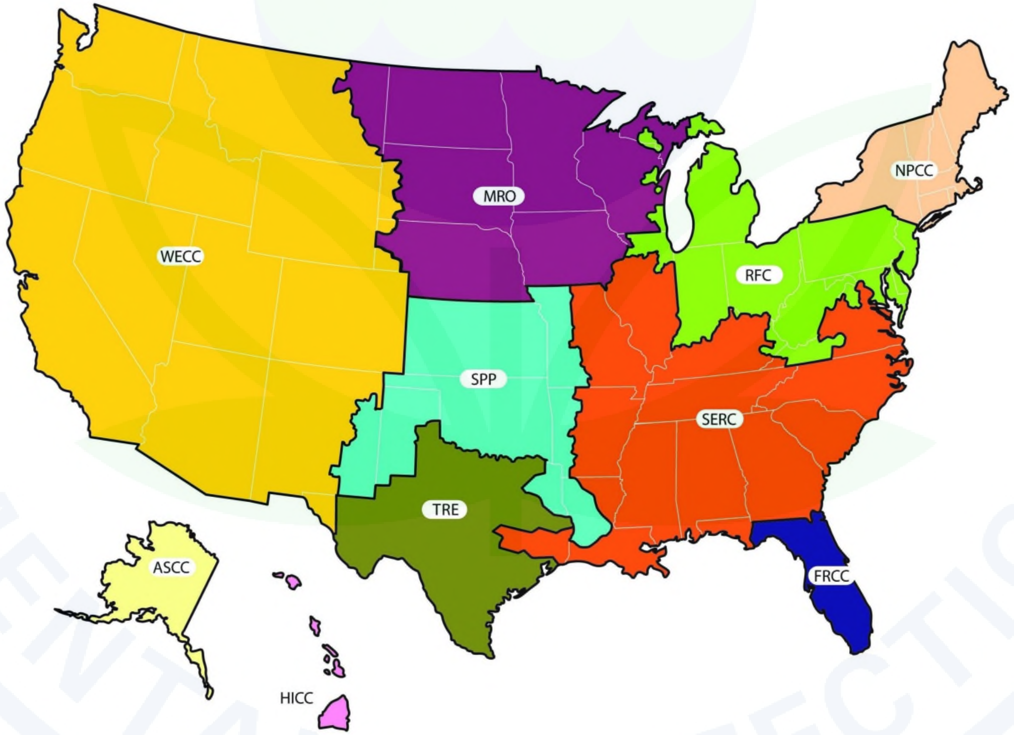
USEPA eGRID2012

8. eGRID2012 NERC Region Resource Mix

NERC region acronym	NERC region name	Nameplate capacity (MW)	Net Generation (MWh)	Generation Resource Mix (percent)										
				Coal	Oil	Gas	Other fossil	Nuclear	Hydro	Biomass	Wind	Solar	Geo-thermal	Other unknown/ purchased fuel
ASCC	Alaska Systems Coordinating Council	2,762.0	6,928,294.5	9.8864	14.9786	52.0863	0.0000	0.0000	22.7335	0.0370	0.2781	0.0000	0.0000	0.0000
FRCC	Florida Reliability Coordinating Council	80,756.1	211,244,527.5	19.4235	0.6443	68.0575	0.6566	8.4594	0.0712	1.7642	0.0000	0.0917	0.0000	0.8317
HICC	Hawaiian Islands Coordinating Council	3,081.6	10,469,268.7	14.6843	71.9229	0.0000	3.4161	0.0000	1.0945	2.7325	3.6128	0.0440	2.4930	0.0000
MRO	Midwest Reliability Organization	72,028.2	232,544,949.0	61.2622	0.2354	5.3532	0.1470	11.4422	5.8749	1.6013	13.8432	0.0000	0.0000	0.2405
NPCC	Northeast Power Coordinating Council	90,299.4	260,500,864.6	3.1082	0.3804	48.5499	1.1282	29.5164	11.9793	3.6349	1.6352	0.0330	0.0000	0.0346
RFC	Reliability First Corporation	279,506.7	916,856,263.3	48.7150	0.4767	18.0186	0.6749	28.7703	0.7023	0.8991	1.6629	0.0446	0.0000	0.0356
SERC	SERC Reliability Corporation	333,238.6	1,088,830,176.0	41.1051	0.4746	28.2648	0.2164	25.5026	2.2712	1.8424	0.2373	0.0104	0.0000	0.0752
SPP	Southwest Power Pool	74,092.2	222,181,961.2	55.3668	0.5558	30.1483	0.1462	3.7289	1.4073	1.0572	7.5232	0.0529	0.0000	0.0133
TRE	Texas Regional Entity	115,787.6	360,221,517.3	30.5073	0.9452	49.0477	0.1204	10.6715	0.1091	0.1977	8.2871	0.0328	0.0000	0.0812
WECC	Western Electricity Coordinating Council	257,842.2	735,740,092.8	26.1944	0.3822	30.2637	0.0777	8.1245	25.7935	1.3181	5.1959	0.4277	2.0773	0.1451
U.S.		**1,309,394.6**	**4,045,517,914.7**	**37.4156**	**0.7034**	**30.2949**	**0.3683**	**19.0169**	**6.7030**	**1.4404**	**3.4476**	**0.1035**	**0.3842**	**0.1221**



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USEPA eGRID2012

9. Year 2012 eGRID Grid Gross Loss (%)

Region	Grid Gross Loss (%)
Eastern	9.17
Western	5.76
ERCOT	7.03
Alaska	8.66
Hawaii	7.69
U.S.	**8.33**

10. eGRID2012 State Emissions and Input Emission Rates

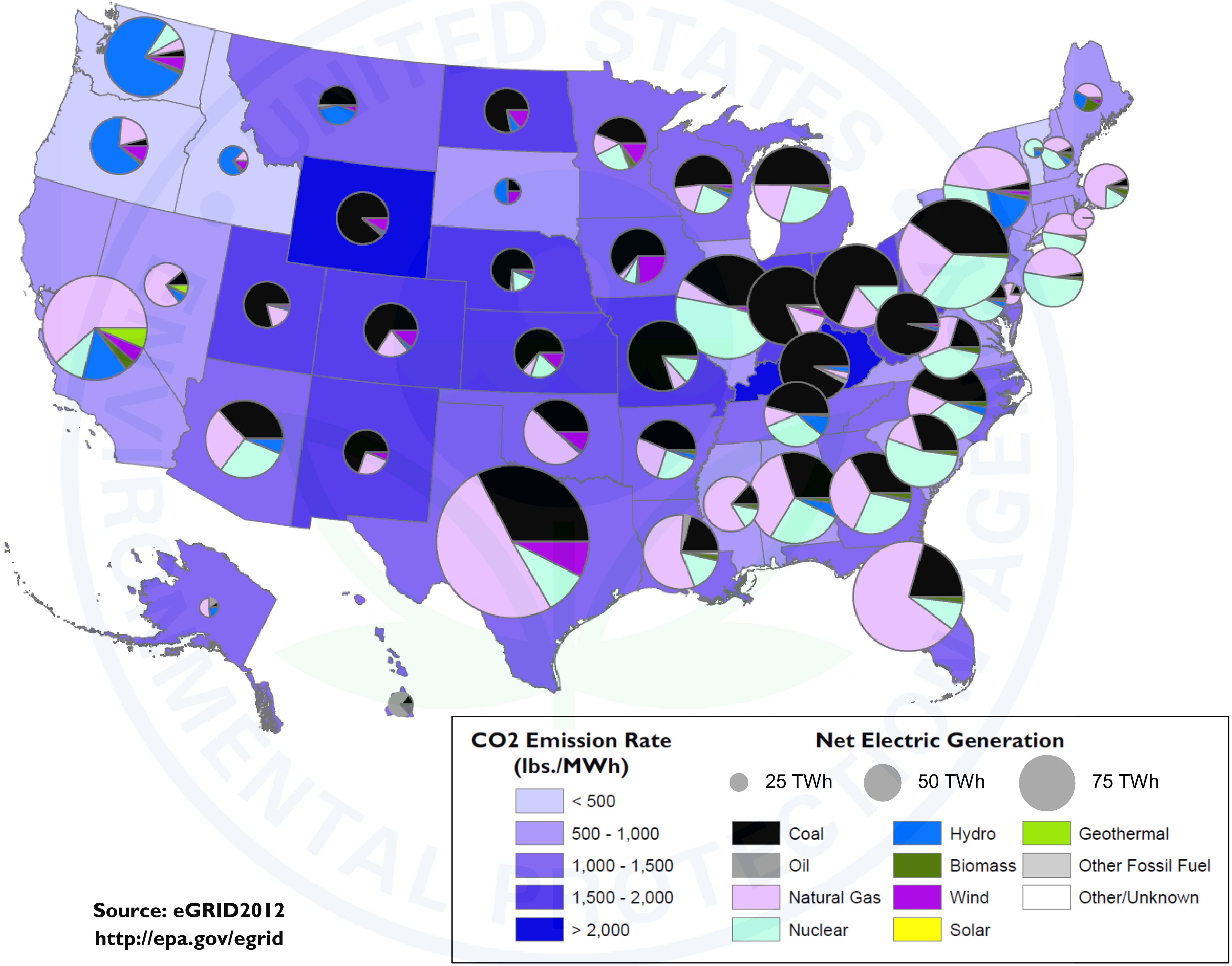
State	Nitrogen oxides (NO_x)				Sulfur dioxide (SO_2)		Carbon dioxide (CO_2)		Methane (CH_4)	Nitrous oxide (N_2O)	Carbon dioxide equivalent (CO_2e)
	Emissions (tons)	Input emission rate (lb/MMBtu)	Ozone season emissions (tons)	Ozone season input emission rate (lb/MMBtu)	Emissions (tons)	Input emission rate (lb/MMBtu)	Emissions (tons)	Input emission rate (lb/MMBtu)	Emissions (lbs)	Emissions (lbs)	Emissions (tons)
AK	11728.39	0.4248	4818.86	0.4385	2384.25	0.0864	3,766,232.7	136.43	170,189.7	46,156.7	3,775,174.0
AL	49346.43	0.1050	24871.13	0.1087	137191.46	0.2918	76,372,600.9	162.46	2,944,703.1	2,041,775.6	76,719,995.5
AR	37359.33	0.1618	17430.69	0.1577	82001.39	0.3552	40,020,363.7	173.36	1,562,067.5	1,244,521.8	40,229,666.2
AZ	49017.27	0.1462	21695.77	0.1345	21055.10	0.0628	58,602,854.8	174.84	1,724,213.4	1,588,514.7	58,867,178.8
CA	18268.77	0.0357	8645.91	0.0376	17622.66	0.0345	57,165,609.7	111.84	6,596,594.8	894,245.8	57,373,339.8
CO	51081.16	0.2232	23006.63	0.2237	41588.09	0.1817	43,843,974.1	191.56	1,063,915.8	1,320,662.8	44,059,835.2
CT	4385.74	0.0600	2208.79	0.0664	19617.59	0.2685	9,332,621.5	127.72	2,009,867.9	264,162.7	9,394,670.3
DC	45.36	0.1309	26.97	0.1447	23.10	0.0667	44,221.2	127.66	2,367.3	350.4	44,300.4
DE	2412.33	0.0599	1264.42	0.0625	3279.30	0.0814	5,424,329.7	134.63	184,373.8	78,240.4	5,438,386.9
FL	72573.66	0.0840	35418.09	0.0857	142273.76	0.1646	125,651,562.0	145.39	8,748,624.0	2,690,783.8	126,160,137.9
GA	38558.19	0.0978	16537.20	0.0815	115772.35	0.2938	65,893,467.8	167.20	2,403,508.2	1,804,357.7	66,198,380.1
HI	15333.42	0.3349	6256.60	0.3291	25283.44	0.5523	7,699,913.5	168.19	880,985.7	199,607.3	7,740,103.0
IA	36946.08	0.1895	16579.87	0.1869	96060.26	0.4926	39,703,157.9	203.60	914,611.3	1,301,654.2	39,914,140.8
ID	649.87	0.0695	229.16	0.0535	776.63	0.0831	885,058.9	94.68	143,366.7	26,625.3	890,691.2
IL	58676.99	0.1186	23716.72	0.1051	167044.47	0.3378	98,493,066.0	199.15	2,284,823.0	3,096,563.1	98,997,023.9
IN	106788.30	0.1917	45383.55	0.1826	280009.33	0.5027	109,335,741.6	196.29	2,645,075.2	3,476,021.9	109,902,298.3
KS	34307.96	0.1944	16700.91	0.1924	32927.81	0.1866	35,312,851.8	200.14	820,114.2	1,113,954.1	35,494,125.9
KY	80461.03	0.1743	36021.19	0.1735	186531.72	0.4041	93,278,019.4	202.09	2,171,107.4	3,115,383.1	93,783,700.4
LA	57596.35	0.1412	27903.34	0.1430	94601.58	0.2319	59,664,031.3	146.23	2,404,235.3	1,163,119.6	59,869,559.3
MA	11083.45	0.0891	3206.11	0.0514	27474.68	0.2209	16,287,831.5	130.98	2,872,403.6	450,408.3	16,387,805.1
MD	17762.97	0.1464	8431.79	0.1314	33960.21	0.2799	22,269,423.2	183.52	1,276,615.9	719,514.9	22,394,352.5
ME	4536.44	0.1007	1581.94	0.0814	9937.79	0.2205	4,056,809.3	90.03	1,951,507.9	277,885.7	4,120,372.4
MI	71122.96	0.1817	31489.24	0.1699	204752.48	0.5230	71,154,710.4	181.74	3,027,319.3	2,247,953.6	71,534,912.4
MN	30490.58	0.1823	12141.84	0.1644	29425.86	0.1760	30,282,137.1	181.10	2,382,107.5	1,112,529.7	30,479,565.6
MO	72777.44	0.1862	35742.01	0.1937	148577.63	0.3801	79,170,368.7	202.56	1,817,465.0	2,562,970.0	79,586,487.8
MS	20935.08	0.1067	11216.50	0.1140	39046.53	0.1989	26,741,087.0	136.23	1,190,006.1	445,222.4	26,822,591.5
MT	18086.67	0.2100	6045.49	0.2077	25992.63	0.3017	17,863,470.4	207.36	400,813.4	583,315.9	17,958,092.9
NC	48807.76	0.1408	23611.01	0.1395	60973.29	0.1760	61,760,659.6	178.23	2,669,867.9	1,970,798.0	62,094,166.9
ND	48794.93	0.3168	20763.77	0.3224	86794.80	0.5636	33,454,500.2	217.22	719,669.8	1,074,306.0	33,628,574.2
NE	28440.45	0.2100	11834.00	0.2027	63840.56	0.4713	28,008,190.7	206.76	628,880.0	918,519.8	28,157,164.6
NH	3598.88	0.0828	1248.76	0.0671	3324.05	0.0765	4,920,946.3	113.24	1,441,319.2	235,472.1	4,972,578.3
NJ	6494.48	0.0500	3744.45	0.0534	12195.68	0.0940	16,860,463.9	129.91	1,436,774.7	287,520.8	16,920,115.8
NM	60305.34	0.3502	27327.22	0.3390	16564.06	0.0962	32,310,828.2	187.64	800,732.6	952,779.6	32,466,916.7
NV	9756.65	0.0822	5038.84	0.0855	4876.18	0.0411	16,119,342.4	135.79	551,201.7	201,328.5	16,156,336.0
NY	23118.18	0.0738	11030.94	0.0705	34220.30	0.1092	38,169,117.1	121.83	3,324,160.7	561,770.0	38,289,788.0
OH	87353.25	0.1584	40997.48	0.1611	348748.84	0.6324	104,821,036.4	190.08	2,645,130.3	3,182,766.7	105,342,121.2
OK	68415.01	0.2087	33251.21	0.1961	78711.17	0.2401	53,328,418.3	162.65	1,589,071.8	1,225,751.6	53,535,095.1
OR	4522.92	0.0726	1369.86	0.0718	14399.06	0.2312	7,896,254.3	126.79	665,314.4	186,427.9	7,932,136.4
PA	133396.37	0.1992	63111.56	0.2074	279451.59	0.4174	118,496,909.6	176.99	5,293,017.5	3,486,416.1	119,092,738.2
RI	762.17	0.0236	380.09	0.0238	35.62	0.0011	3,768,286.1	116.71	148,274.5	14,973.0	3,772,163.8
SC	21243.79	0.0974	9568.15	0.0946	54472.96	0.2497	37,176,847.9	170.41	1,684,240.0	1,197,347.9	37,380,121.3
SD	10849.69	0.6564	4559.75	0.6153	12403.15	0.7504	3,358,271.8	203.17	77,327.0	107,308.6	3,375,716.5
TN	23869.57	0.1084	11456.37	0.1038	69210.21	0.3143	42,463,377.1	192.82	1,126,716.7	1,342,973.9	42,683,368.6
TX	150149.67	0.0946	70889.70	0.0906	410195.38	0.2585	258,352,875.5	162.84	7,671,865.4	5,623,891.9	259,305,005.5
UT	51961.45	0.2804	22287.72	0.2713	23116.61	0.1248	35,475,233.4	191.45	871,733.2	1,103,711.6	35,655,461.9
VA	24925.47	0.1296	12319.67	0.1262	41121.56	0.2139	28,892,649.6	150.28	2,438,306.3	811,218.6	29,043,990.7
VT	196.71	0.0742	101.11	0.0934	60.10	0.0227	13,780.1	5.20	341,084.3	45,542.0	24,420.5
WA	6848.53	0.1335	2434.22	0.1296	6357.58	0.1239	7,342,211.5	143.10	892,265.5	280,845.7	7,394,968.1
WI	27372.34	0.1165	12922.98	0.1146	71901.90	0.3061	43,242,292.9	184.08	1,807,488.0	1,401,191.1	43,478,456.1
WV	50578.41	0.1450	23312.06	0.1482	88849.23	0.2546	72,352,646.6	207.36	1,622,530.6	2,427,499.4	72,745,945.5
WY	49592.64	0.1984	19613.45	0.1939	50386.32	0.2015	52,023,827.5	208.08	1,159,613.9	1,719,605.7	52,302,542.3
U.S.	**1,913,686.86**	**0.1427**	**871,745.09**	**0.1379**	**3,817,422.30**	**0.2847**	**2,298,924,483.4**	**171.43**	**96,199,568.7**	**64,226,468.3**	**2,309,886,780.4**

11. eGRID2012 State Resource Mix

State	Nameplate capacity (MW)	Net generation (MWh)	Generation Resource Mix (percent)										
			Coal	Oil	Gas	Other Fossil	Nuclear	Hydro	Biomass	Wind	Solar	Geo-thermal	Other unknown/ purchased fuel
AK	2,762.0	6,928,294.5	9.8864	14.9786	52.0863	0.0000	0.0000	22.7335	0.0370	0.2781	0.0000	0.0000	0.0000
AL	36,284.1	153,105,217.0	29.7880	0.0716	36.3835	0.1160	26.6753	4.8563	2.1094	0.0000	0.0000	0.0000	0.0000
AR	18,689.2	65,005,677.9	43.7361	0.0501	26.3322	0.0468	23.8335	3.4472	2.5542	0.0000	0.0000	0.0000	0.0000
AZ	35,774.8	110,614,113.4	36.2663	0.0379	27.3880	0.0000	28.8697	6.1442	0.2125	0.2255	0.8559	0.0000	0.0000
CA	97,737.5	199,189,655.8	0.6317	0.8499	60.0661	0.0908	9.2913	13.7605	3.1494	4.8624	0.6829	6.2850	0.3299
CO	16,952.1	52,547,910.6	65.6944	0.0210	20.0283	0.0000	0.0000	2.3840	0.1104	11.3458	0.3108	0.0000	0.1054
CT	10,902.7	35,557,337.4	0.2696	0.3024	46.5113	2.0675	48.0292	0.8855	1.9346	0.0000	0.0000	0.0000	0.0000
DC	860.8	71,786.8	0.0000	13.0929	86.9071	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DE	3,999.4	8,633,823.3	16.4782	3.0733	78.9306	0.0000	0.0000	0.0000	1.2145	0.0421	0.2613	0.0000	0.0000
FL	83,533.4	221,099,929.6	20.0300	0.6223	67.7068	0.6277	8.0823	0.0681	1.9807	0.0000	0.0876	0.0000	0.7946
GA	48,472.3	122,014,744.3	33.3690	0.3118	34.6502	0.0423	27.8176	1.1461	2.6617	0.0000	0.0013	0.0000	0.0000
HI	3,081.6	10,469,268.7	14.6843	71.9229	0.0000	3.4161	0.0000	1.0945	2.7325	3.6128	0.0440	2.4930	0.0000
IA	18,509.8	56,602,145.7	62.4205	0.1883	3.4290	0.0226	7.6799	1.3536	0.2441	24.6620	0.0000	0.0000	0.0000
ID	5,388.7	15,499,089.3	0.4951	0.0001	12.2470	0.0000	0.0000	70.5874	3.5439	12.1979	0.0000	0.4818	0.4469
IL	59,211.6	197,522,001.0	40.9204	0.1015	5.6647	0.1035	48.8054	0.0563	0.3115	3.8899	0.0155	0.0000	0.1314
IN	37,915.2	114,878,967.3	80.6452	1.1142	12.5967	1.8951	0.0000	0.3774	0.2932	2.7943	0.0000	0.0000	0.2840
KS	15,927.4	44,286,624.6	63.1851	0.0776	6.4575	0.0000	18.7075	0.0235	0.1295	11.4193	0.0000	0.0000	0.0000
KY	28,259.8	89,957,452.2	91.9999	1.7081	3.2875	0.0090	0.0000	2.6254	0.3700	0.0000	0.0000	0.0000	0.0000
LA	30,604.9	103,347,602.4	20.7280	2.9317	56.6231	1.2867	15.1519	0.6579	2.3513	0.0000	0.0000	0.0000	0.2693
MA	16,284.9	36,198,121.5	5.9034	0.4821	68.1593	2.4200	16.1874	1.6712	4.8471	0.2477	0.0818	0.0000	0.0000
MD	14,595.8	37,808,347.2	42.8074	0.3632	13.0788	1.0541	35.9161	4.3814	1.4925	0.8508	0.0557	0.0000	0.0000
ME	5,527.8	14,420,135.4	0.3139	0.5809	41.9115	2.2748	0.0000	25.8847	22.3179	6.0919	0.0000	0.0000	0.6245
MI	34,036.7	108,166,077.4	49.1249	0.3006	20.1064	0.6499	25.9043	0.4082	2.4593	1.0463	0.0000	0.0000	0.0000
MN	18,009.4	52,193,624.2	43.5355	0.0567	13.5806	0.5017	22.8836	1.0749	3.5404	14.5907	0.0000	0.0000	0.2358
MO	24,141.3	91,804,321.4	79.2719	0.0732	6.7293	0.0215	11.6752	0.8145	0.0586	1.3559	0.0000	0.0000	0.0000
MS	19,469.2	54,584,295.2	13.2125	0.0313	70.6245	0.0000	13.3667	0.0000	2.7649	0.0000	0.0000	0.0000	0.0000
MT	6,693.6	27,795,017.1	50.3234	1.6801	1.6694	0.0000	0.0000	40.5953	0.0000	4.5044	0.0000	0.0000	1.2275
NC	39,312.0	116,971,226.6	43.5425	0.1524	16.5015	0.1204	33.6712	3.4585	2.1924	0.0000	0.0953	0.0000	0.2659
ND	7,390.6	36,125,158.9	78.1017	0.1884	0.0601	0.0029	0.0000	6.8574	0.0153	14.6007	0.0000	0.0000	0.1736
NE	9,084.1	34,200,814.9	73.1539	0.0660	2.2517	0.0000	16.9633	3.6755	0.1846	3.7050	0.0000	0.0000	0.0000
NH	4,720.9	19,264,434.9	6.5806	0.1124	36.5944	0.3283	42.5093	6.6931	6.0986	1.0833	0.0000	0.0000	0.0000
NJ	23,680.2	65,232,564.1	2.9088	0.4911	43.3601	0.7771	50.7572	-0.2383	1.5070	0.0177	0.4192	0.0000	0.0000
NM	9,965.1	36,635,909.3	68.2228	0.1260	24.0175	0.0000	0.0000	0.6082	0.0389	6.0754	0.9111	0.0000	0.0000
NV	17,929.0	35,142,774.0	11.6082	0.0537	72.9785	0.0208	0.0000	6.9443	0.0540	0.3665	1.2842	6.6560	0.0339
NY	48,055.8	135,662,526.5	3.3547	0.4277	43.8310	0.6904	30.0559	17.8021	1.6067	2.1927	0.0389	0.0000	0.0000
OH	39,660.3	129,741,418.3	65.9686	0.9881	17.4690	0.7483	13.1700	0.3192	0.5523	0.7596	0.0249	0.0000	0.0000
OK	25,816.9	77,757,667.7	37.6834	0.0138	50.1852	0.0125	0.0000	1.3231	0.4675	10.3146	0.0000	0.0000	0.0000
OR	18,972.1	60,612,559.4	4.3462	0.0098	19.1787	0.0671	0.0000	65.0148	1.3881	9.9579	0.0106	0.0268	0.0000
PA	54,685.0	223,416,431.4	39.0071	0.1614	23.7508	0.6207	33.6477	0.8050	1.0417	0.9528	0.0129	0.0000	0.0000
RI	2,052.2	8,309,035.9	0.0000	0.2151	98.5072	0.0000	0.0000	0.0513	1.2097	0.0166	0.0000	0.0000	0.0000
SC	26,596.0	96,755,682.3	29.3483	0.1120	14.8124	0.1098	52.8603	0.5418	2.2153	0.0000	0.0000	0.0000	0.0000
SD	4,432.9	12,017,722.0	24.2871	0.0476	1.7815	0.0000	0.0000	49.6307	0.0000	24.2531	0.0000	0.0000	0.0000
TN	25,710.6	77,385,936.5	45.7697	0.1867	10.4349	0.0181	32.4375	10.0878	1.0038	0.0614	0.0000	0.0000	0.0000
TX	135,365.8	429,697,350.7	32.1604	0.8878	49.7795	0.1567	8.9461	0.1360	0.3920	7.4459	0.0275	0.0000	0.0681
UT	8,826.7	39,400,420.8	78.1695	0.1110	16.6929	0.0113	0.0000	1.8979	0.1516	1.7866	0.0041	0.8493	0.3259
VA	29,875.0	70,739,234.7	20.0466	0.5145	35.3944	0.5406	40.6046	-0.4549	3.3541	0.0000	0.0000	0.0000	0.0000
VT	1,276.0	6,568,121.0	0.0000	0.0521	0.0387	0.0000	75.9629	16.8877	5.3776	1.6275	0.0535	0.0000	0.0000
WA	32,149.1	116,834,423.7	3.2208	0.3698	4.6541	0.0568	7.9888	76.6109	1.3833	5.6479	0.0007	0.0000	0.0669
WI	21,870.5	63,742,909.9	51.3915	0.4982	18.1071	0.0702	22.4332	2.3881	2.6176	2.4435	0.0000	0.0000	0.0506
WV	18,416.3	73,413,404.2	95.7147	0.1954	0.3309	0.0431	0.0000	1.9498	0.0144	1.7518	0.0000	0.0000	0.0000
WY	9,925.5	49,588,606.1	87.5438	0.1263	1.0343	0.5479	0.0000	1.8018	0.0000	8.8107	0.0000	0.0000	0.1351
U.S.	**1,309,394.6**	**4,045,517,914.7**	**37.4156**	**0.7034**	**30.2949**	**0.3683**	**19.0169**	**6.7030**	**1.4404**	**3.4476**	**0.1035**	**0.3842**	**0.1221**

NWN/501
Summers/13

eGRID2012 Generation by Fuel Type and CO_2 Emission Rates



Source: eGRID2012
http://epa.gov/egrid

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UM 1744
Carbon Emission Reduction Program
Combined Heat & Power (CHP)

Investment Analysis and Portfolio Management, Chapter 13 – "*Computing Bond Yields*"

October 16, 2015

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5. Complete the information requested for each of the following $1,000 face value, zero coupon bonds, assuming semiannual compounding.

Bond	Maturity (Years)	Yield (Percent)	Price ($)
A	20	12	?
B	?	8	601
C	9	?	350

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Chapter 16

The Analysis and Valuation of Bonds

This chapter answers the following questions:

- How do you determine the value of a bond based on the present value formula?
- What are the alternative bond yields that are important to investors?
- How do you compute the following major yields on bonds: current yield, yield to maturity, yield to call, and compound realized (horizon) yield?
- What are spot rates and forward rates and how do you calculate these rates from a yield to maturity curve?
- What is the spot rate yield curve and forward rate curve?
- How and why do you use the spot rate curve to determine the value of a bond?
- What are the alternative theories that attempt to explain the shape of the term structure of interest rates?
- What factors affect the level of bond yields at a point in time?
- What economic forces cause changes in bond yields over time?
- When yields change, what characteristics of a bond cause differential price changes for individual bonds?
- What is meant by the duration of a bond, how do you compute it, and what factors affect it?
- What is modified duration and what is the relationship between a bond's modified duration and its volatility?
- What is effective duration and when is it useful?
- What is the convexity for a bond, how do you compute it, and what factors affect it?
- Under what conditions is it necessary to consider both modified duration and convexity when estimating a bond's price volatility?
- What happens to the duration and convexity of bonds that have embedded call options?

In this chapter, we apply the valuation principles that were introduced in Chapter 13 to the valuation of bonds. This chapter is concerned with how one goes about finding the value of bonds using the traditional single yield to maturity rate and using multiple spot rates. We will also come to understand the several measures of yields for bonds. It also is important to understand why these bond values and yields change over time. To do this, we begin with a review of value estimation for bonds using the present value model introduced in Chapter 13. This background on valuation allows us to understand and compute the expected rates of return on bonds, which are their yields. We need to understand how to measure alternative yields on bonds because they are very important to bond investors.

After mastering the measurement of bond yields, we consider what factors influence the level of bond yields and what economic forces cause changes in yields over time. This is followed by a consideration of the alternative shapes of the yield curve and the alternative theories that explain changes in its shape. We discuss the effects of various characteristics and indenture provisions that affect the required returns and, therefore, the value of specific bond issues. This includes factors such as time to maturity, coupon, callability, and sinking funds.

We return to the consideration of bond value and examine the characteristics that cause different changes in a bond's price. When yields change, all bond prices do not change in the same way. An understanding of the factors that affect the price changes for bonds has become more important during the past several decades because the price volatility of bonds has increased substantially. Before 1950, the yields on bonds were fairly low and both yields and prices were stable. In this environment, bonds were considered a very safe investment and most investors in bonds intended to hold them to maturity. During the last several decades, however, the level of interest rates has increased substantially because of inflation, and interest rates have also become more volatile because of changes in the rate of inflation and monetary policy. As a result, bond prices and rates of return on bonds have been much more volatile and the rates of return on bond investments have increased. Although this increase in interest rate volatility has affected all bonds, the impact is more significant on bonds with embedded options such as call features.

THE FUNDAMENTALS OF BOND VALUATION

The value of bonds can be described in terms of dollar values or the rates of return they promise under some set of assumptions. In this section, we describe both the present value model, which computes a specific value for the bond using a single discount value, and the yield model, which computes the promised rate of return based on the bond's current price.

The Present Value Model

In our introduction to valuation theory in Chapter 13, we saw that the value of a bond (or any asset) equals the present value of its expected cash flows. The cash flows from a bond are the periodic interest payments to the bondholder and the repayment of principal at the maturity of the bond. Therefore, the value of a bond is the present value of the semiannual interest payments plus the present value of the principal payment. Notably, the standard technique is to use a single interest rate discount factor, which is the required rate of return on the bond. We can express this in the following present value formula that assumes semiannual compounding:[1]

$$P_m = \sum_{t=1}^{2n} \frac{C_i/2}{(1 + i/2)^t} + \frac{P_p}{(1 + i/2)^{2n}}$$

where

P_m = the current market price of the bond
n = the number of years to maturity
C_i = the annual coupon payment for bond i
i = the prevailing yield to maturity for this bond issue
P_p = the par value of the bond

The value computed indicates what an investor would be willing to pay for this bond to realize a rate of return that takes into account expectations regarding the RFR, the expected rate of inflation, and the risk of the bond. The standard valuation technique assumes

[1] Almost all U.S. bonds pay interest semiannually so it is appropriate to use semiannual compounding wherein you cut the annual coupon rate in half and double the number of periods. To be consistent, you should also use semiannual compounding for the principal payment of a coupon bond or even a zero coupon bond. All our present value calculations assume semiannual compounding.

NWN/502
Summers/3

holding the bond to the maturity of the obligation. In this case, the number of periods would be the number of years to the maturity of the bond (referred to as its *term to maturity*). In such a case, the cash flows would include all the periodic interest payments and the payment of the bond's par value at the maturity of the bond.

We can demonstrate this formula using an 8 percent coupon bond that matures in 20 years with a par value of $1,000. This calculation implies that an investor who holds this bond to maturity will receive $40 every six months (one half of the $80 coupon) for 20 years (40 periods) and $1,000 at the maturity of the bond in 20 years. If we assume a prevailing yield to maturity for this bond of 10 percent (the market's required rate-of-return on the bond), the value for the bond using the above equation would be:

$$P_m = \sum_{t=1}^{40} \frac{80/2}{(1 + .10/2)^t} + \frac{\$1,000}{(1 + .10/2)^{40}}$$

We know that the first term is the present value of an annuity of $40 every six months for 40 periods at 5 percent, while the second term is the present value of $1,000 to be received in 40 periods at 5 percent. This can be summarized as follows:

Present value of interest payments: $40 × 17.1591	=	$686.36
Present value of principal payment $1,000 × .1420	=	142.00
Total value of bond at 10%		$828.36

As expected, the bond will be priced at a discount to its par value because the market's required rate of return of 10 percent is greater than the bond's coupon rate, i.e., $828.36 or 82.836 percent of par.

Alternatively, if the market's required rate was 6 percent, the value would be computed the same way except we would compute the present value of the annuity at 3 percent for 40 periods and the present value of the principal at 3 percent for 40 periods as follows:

Present value of interest payments: $40 × 23.1148	=	$924.59
Present value of principal payment $1,000 × .3066	=	306.60
Total value of bond at 6%		$1,231.19

Because the bond's discount rate is lower than its coupon, the bond would sell at a premium above par value—i.e., $1,231.19 or 123.119 of par.

THE PRICE-YIELD CURVE When you know the basic characteristics of a bond in terms of its coupon, maturity, and par value, the only factor that determines its value (price) is the market discount rate—its required rate of return. As shown above, as we increase the required rate, the price declines. It is possible to demonstrate the specific relationship between the price of a bond and its yield by computing the bond's price at a range of yields as shown in Table 16.1.

NWN/502
Summers/4

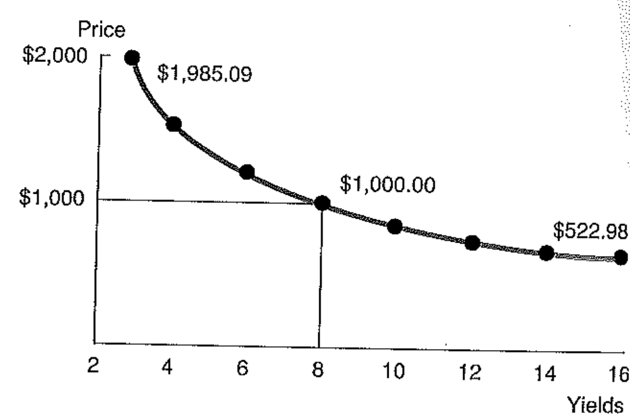
TABLE 16.1

PRICE-YIELD RELATIONSHIP FOR A 20-YEAR, 8 PERCENT COUPON BOND ($1,000 PAR VALUE)

Required Yield	Price of Bond
2	$1,985.09
4	1,547.12
6	1,231.19
8	1,000.00
10	828.36
12	699.05
14	600.07
16	522.98

FIGURE 16.1

THE PRICE-YIELD CURVE FOR A 20-YEAR, 8 PERCENT COUPON BOND



A graph of this relationship between the required return (yield) on the bond and its price is referred to as the price-yield curve as shown in Figure 16.1. Besides demonstrating that price moves inverse to yield, it shows three other important points:

1. When the yield is below the coupon rate, the bond will be priced at a **premium** to its par value.
2. When the yield is above the coupon rate, the bond will be priced at a **discount** to its par value.
3. The price-yield relationship is not a straight line; rather, it is *convex*. As yields decline, the price increases at an increasing rate, and as the yield increases, the price declines at a declining rate. This concept of a convex price-yield curve is referred to as *convexity* and will be discussed further in a later section.

THE YIELD MODEL

Instead of determining the value of a bond in dollar terms, investors often price bonds in terms of **yields**—the promised rates of return on bonds under certain assumptions. Thus far, we have used cash flows and our required rate of return to compute an estimated value for the bond. To compute an expected yield, we use the current market price (P_m) and the expected cash flows to *compute the expected yield on the bond*. We can express this approach using the same present value model. The difference is that in the equation on page 526, it was assumed that we knew the appropriate discount rate (the required rate of return), and we computed the estimated value (price) of the bond. In this case, it is assumed that we know the price of the bond and we compute the discount rate (yield) that will give us the current market price (P_m).

$$P_m = \sum_{t=1}^{2n} \frac{C_i/2}{(1+i/2)^t} + \frac{P_p}{(1+i/2)^{2n}}$$

where the variables are the same as previously, except

i = the discount rate that will discount the expected cash flows to equal the current market price of the bond

This i value gives the expected ("promised") yield of the bond under various assumptions to be noted, assuming you pay the price P_m. We will discuss several types of bond yields that arise from the assumptions of the valuation model in the next section.

Approaching the investment decision stating the bond's value as a yield figure rather than a dollar amount, you consider the relationship of the computed bond yield to your required rate of return on this bond. If the computed bond yield is equal to or greater than your required rate of return, you should buy the bond; if the computed yield is less than your required rate of return, you should not buy the bond.

These approaches to pricing bonds and making investment decisions are similar to the two alternative approaches by which firms make investment decisions. We referred to one approach, the net present value (NPV) method, in Chapter 13. With the NPV approach, you compute the present value of the net cash flows from the proposed investment at your cost of capital and subtract the present value cost of the investment to get the net present value (NPV) of the project. If this NPV is positive, you consider accepting the investment; if it is negative, you reject it. This is basically the way we compared the value of an investment to its market price.

The second approach is to compute the **internal rate of return (IRR)** on a proposed investment project. The IRR is the discount rate that equates the present value of cash outflows for an investment with the present value of its cash inflows. You compare this discount rate, or IRR (which is also the expected rate of return on the project), to your cost of capital, and accept any investment proposal with an IRR equal to or greater than your cost of capital. We do the same thing when we price bonds on the basis of yield. If the expected yield on the bond (yield to maturity, yield to call, or horizon yield) is equal to or exceeds your required rate of return on the bond, you should invest in it; if the expected yield is less than your required rate of return on the bond, you should not invest in it.

COMPUTING BOND YIELDS

Bond investors traditionally have used five yield measures for the following purposes:

Yield Measure	Purpose
Nominal yield	Measures the coupon rate.
Current yield	Measures the current income rate.
Promised yield to maturity	Measures the expected rate of return for bond held to maturity.
Promised yield to call	Measures the expected rate of return for bond held to first call date.
Realized (horizon) yield	Measures the expected rate of return for a bond likely to be sold prior to maturity. It considers specific reinvestment assumptions and an estimated sales price. It also can measure the actual rate of return on a bond during some past period of time.

Nominal and current yields are mainly descriptive and contribute little to investment decision making. The last three yields are all derived from the present value model as described in the equation on page 529.

When we present the last three yields based on the present value model, we consider two calculation techniques. First, we consider a fairly simple calculation for the approximate values for each of these yields to provide reasonable estimates. Second, we use the present value model to get accurate values. We provide both techniques because an exact answer with the present value model requires several calculations. In some cases, the approximate yield value is adequate.

To measure an expected realized yield (also referred to as the horizon yield), a bond investor must estimate a bond's future selling price. Following our presentation of bond yields, we present the procedure for finding these prices. We conclude the section by examining the yields on tax-free bonds.

NOMINAL YIELD **Nominal yield** is the coupon rate of a particular issue. A bond with an 8 percent coupon has an 8 percent nominal yield. This provides a convenient way of describing the coupon characteristics of an issue.

CURRENT YIELD **Current yield** is to bonds what dividend yield is to stocks. It is computed as

$$CY = C_i / P_m$$

where

CY = the current yield on a bond
C_i = the annual coupon payment of bond i
P_m = the current market price of the bond

Because this yield measures the current income from the bond as a percentage of its price, it is important to income-oriented investors who want current cash flow from their investment portfolios. An example of such an investor would be a retired person who lives on this investment income. Current yield has little use for most other investors who are interested in total return because it excludes the important capital gain or loss component.

PROMISED YIELD TO MATURITY **Promised yield to maturity** is the most widely used bond yield figure because it indicates the fully compounded rate of return promised to an investor who buys the bond at prevailing prices, *if two assumptions hold true*. Specifically, the *promised* yield to maturity will be equal to the investor's *realized* yield *if* these assumptions are met. The first assumption is that the investor holds the bond to maturity. This assumption gives this value its shortened name, *yield to maturity* (YTM). The second assumption is implicit in the present value method of computation. Referring back to the equation on page 529, recall that it relates the current market price of the bond to the present value of all cash flows as follows:

$$P_m = \sum_{t=1}^{2n} \frac{C_i/2}{(1+i/2)^t} + \frac{P_p}{(1+i/2)^{2n}}$$

To compute the YTM for a bond, we solve for the rate i that will equate the current price (P_m) to all cash flows from the bond to maturity. As noted, this resembles the computation of the internal rate of return (IRR) on an investment project. Because it is a present value-based computation, it implies a reinvestment rate assumption because it discounts the cash flows. That is, the equation assumes that *all interim cash flows (interest payments) are reinvested at the computed YTM*. That is why this is referred to as a *promised* YTM because the bond will provide this computed YTM *only if* you meet its conditions:

1. You hold the bond to maturity.
2. You reinvest all the interim cash flows at the computed YTM rate.

If a bond promises an 8 percent YTM, you must reinvest coupon income at 8 percent to realize that promised return. If you spend (do not reinvest) the coupon payments or if you cannot find opportunities to reinvest these coupon payments at rates as high as its promised YTM, then the actual realized yield you earn will be less than the promised yield to maturity. As will be demonstrated in the section on realized return, if you can reinvest at rates above the YTM, your realized (horizon) return will be greater than the promised YTM. The income earned on this reinvestment of the interim interest payments is referred to as **interest-on-interest**.[2]

The impact of the reinvestment assumption (i.e., the interest-on-interest earnings) on the actual return from a bond varies directly with the bond's coupon and maturity. A higher coupon and/or a longer term to maturity will increase the loss in value from failure to reinvest at the YTM. Therefore, a higher coupon or a longer maturity makes the reinvestment assumption more important.

Figure 16.2 illustrates the impact of interest-on-interest for an 8 percent, 25-year bond bought at par to yield 8 percent. If you invested $1,000 today at 8 percent for 25 years and reinvested all the coupon payments at 8 percent, you would have approximately $7,100 at the end of 25 years. We will refer to this money that you have at the end of your investment horizon as your **ending-wealth value**. To prove that you would have an ending-wealth value of $7,100 look up the compound interest factor for 8 percent for 25 years (6.8493) or 4 percent for 50 periods (which assumes semiannual compounding and is 7.1073). In the case of U.S. bonds, the semiannual compounding is the appropriate procedure because almost all bonds pay interest every six months.

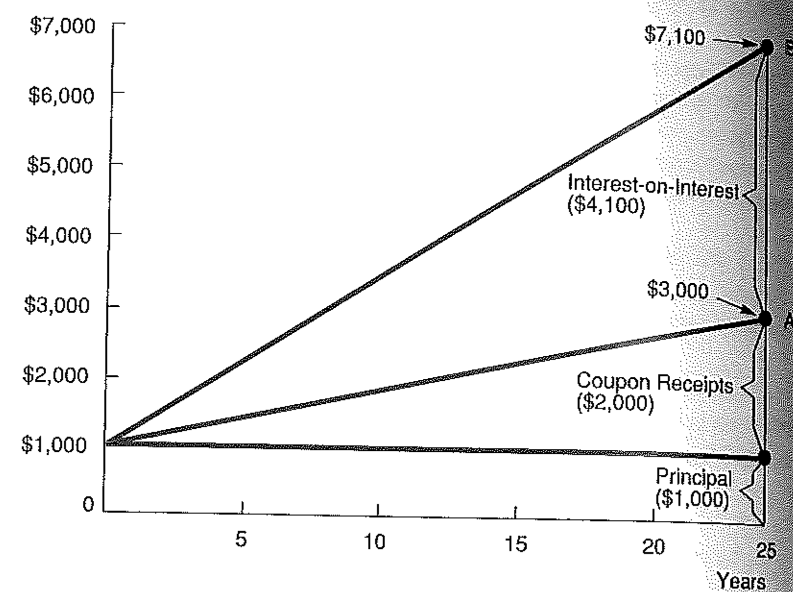
Figure 16.2 shows that this $7,100 is made up of $1,000 principal return, $2,000 of coupon payments over the 25 years ($80 a year for 25 years), and $4,100 in interest earned on the semiannual coupon payments reinvested at 4 percent semiannually. If you had never reinvested any of the coupon payments, you would have an ending-wealth value of only $3,000. This ending-wealth value of $3,000 derived from the beginning investment of $1,000 gives you an actual (realized) yield to maturity of only 4.5 percent. That is, the rate that will discount $3,000 back to $1,000 in 25 years is 4.5 percent. Reinvesting the coupon payments at some rate between 0 and 8 percent would cause your ending-wealth position to be above $3,000 and below $7,100; therefore, your actual rate of return would be somewhere between 4.5 percent and 8 percent. Alternatively, if you managed to reinvest the coupon payments at rates consistently above 8 percent, your ending-wealth position would be above $7,100, and your realized (horizon) rate of return would be above 8 percent.

Interestingly, during periods of very high interest rates, you often hear investors talk about "locking in" high yields. Many of these people are subject to **yield illusion** because they do not realize that attaining the high promised yield requires that they reinvest all the coupon payments at the very high promised yields. For example, if you buy a 20-year bond with a promised yield to maturity of 15 percent, you will actually realize the promised

[2]This concept is developed in Sidney Homer and Martin L. Leibowitz, *Inside the Yield Book* (Englewood Cliffs, NJ: Prentice-Hall, 1972), Chapter 1.

NWN/502
Summers/6

FIGURE 16.2 THE EFFECT OF INTEREST-ON-INTEREST ON TOTAL REALIZED RETURN



Promised yield at time of purchase: 8.00%
Realized yield over the 25-year investment horizon with no coupon reinvestment (*A*): 4.50%
Realized yield over the 25-year horizon with coupons reinvested at 8% (*B*): 8.00%

15 percent yield *only* if you are able to reinvest all the coupon payments at 15 percent over the next 20 years.

COMPUTING THE PROMISED YIELD TO MATURITY The promised yield to maturity can be computed in two ways: finding an approximate annual yield, or using the present value model with semiannual compounding. The present value model gives an investor a more accurate result, and is the technique used by investment professionals.

The approximate promised yield (APY) measure is easy to calculate:

$$APY = \frac{C_i + \frac{P_p - P_m}{n}}{\frac{P_p + P_m}{2}}$$

$$= \frac{\text{Coupon + Annual Straight-Line Amortization of Capital Gain or Loss}}{\text{Average Investment}}$$

where variables are as defined earlier. This approximate value for the promised yield to maturity assumes interest is compounded annually, and it does not require the multiple computations of the present value model. An 8 percent bond with 20 years remaining to maturity and a current price of $900 has an approximate yield of 8.95 percent:

$$APY = \frac{80 + \frac{1000 - 900}{20}}{\frac{1000 + 900}{2}} = \frac{80 + 5}{950}$$

$$= 8.95\%.$$

The present value model provides a more accurate yield to maturity value. Again, the equation on page 529 shows the promised yield valuation model:

$$P_m = \sum_{t=1}^{2n} \frac{C_i/2}{(1 + i/2)^t} + \frac{P_p}{(1 + i/2)^{2n}}$$

All variables are as described previously. This model is more accurate than the approximate promised yield model, but also is more complex because the solution requires iteration. The present value equation is a variation of the internal rate of return (IRR) calculation where we want to find the discount rate, *i*, that will equate the present value of the stream of coupon receipts (C_i) and principal value (P_p) with the current market price of the bond (P_m). Using the prior example of an 8 percent, 20-year bond, priced at $900, the equation gives us a semiannual promised yield to maturity of 4.545 percent, which implies an annual promised YTM of 9.09 percent.[3]

$$900 = 40 \sum_{t=1}^{40} \left(\frac{1}{(1.04545)^t}\right) + 1000 \left(\frac{1}{(1.04545)^{40}}\right)$$

$$= 40(18.2574) + 1000(.1702)$$
$$= 900.$$

The values for $1/(1 + i)$ were taken from the present value interest factor tables in the appendix at the back of the book using interpolation.

Comparing the results of this equation with those of the approximate promised yield computation, you find a variation of 14 basis points (8.95 percent vs. 9.09 percent). As a rule, the approximate promised yield tends to understate the present value promised yield for issues selling below par value (i.e., trading at a discount) and to overstate the promised yield for a bond selling at a premium. The size of the differential varies directly with the length of the holding period. Although the estimated yield value differs, the rankings of yields estimated using the APY formula will generally be identical to those determined by the present value method.

YTM FOR A ZERO COUPON BOND In several instances we have discussed the existence of zero coupon bonds that only have the one cash inflow at maturity. This single cash flow means that the calculation of YTM is substantially easier as shown by the following example:

Assume a zero coupon bond, maturing in 10 years with a maturity value of $1,000 selling for $311.80. Because you are dealing with a zero coupon bond, there is only the one cash flow from the principal payment at maturity. Therefore, you simply need to determine what is the discount rate that will discount $1,000 to equal the current market price of $311.80 in 20 periods (10 years of semiannual payments). The equation is as follows:

$$\$311.80 = \frac{\$1000}{(1 + i)^{20}}$$

[3] You will recall from your corporate finance course that you start with one rate (e.g., 9 percent or 4.5 percent semiannual) and compute the value of the stream. In this example, the value would exceed $900, so you would select a higher rate until you had a present value for the stream of cash flows of less than $900. Given the discount rates above and below the true rate, you would do further calculations or interpolate between the two rates to arrive at the correct discount rate that would give you a value of $900.

You will see that $i = 6$ percent, which implies an annual rate of 12 percent. For future reference, this yield also is referred to as the 10-year spot rate, which is the discount rate for a single cash flow to be received in 10 years.

Promised Yield to Call

Although investors use promised YTM to value most bonds, they must estimate the return on certain callable bonds with a different measure—the **promised yield to call (YTC)**. Whenever a bond with a call feature is selling for a price above par (i.e., at a premium) equal to or greater than its par value plus one year's interest, a bond investor should consider valuing the bond in terms of YTC rather than YTM. This is because the market place uses the lowest, most conservative yield measure in pricing a bond. When bonds are trading at or above a specified **crossover price,** which is approximately the bond's call price plus a small premium that increases with time to call, the yield to call will provide the lowest yield measure.[4] The crossover price is important because at this price the YTM and the YTC are equal—this is the *crossover yield*. When the bond rises to this price above par, the computed YTM becomes low enough that it would be profitable for the issuer to call the bond and finance the call by selling a new bond at this prevailing market interest rate. Therefore, the YTC measures the promised rate of return the investor will receive from holding this bond until it is retired at the first available call date, that is, at the end of the deferred call period. Note that if an issue has multiple call dates at different prices (the call price will decline for later call dates), it will be necessary to compute which of these scenarios provides the lowest yield—this is referred to as *yield to worst*. Investors must consider computing the YTC for their bonds after a period when numerous high-yielding, high-coupon bonds have been issued. Following such a period, interest rates will decline, bond prices will rise, and the high coupon bonds will subsequently have a high probability of being called.

COMPUTING PROMISED YIELD TO CALL Again, there are two methods for computing the promised yield to call: the approximate method and the present value method. Both methods assume that you hold the bond until the first call date. The present value method also assumes that you reinvest all coupon payments at the YTC rate.

Yield to call is calculated using variations of the equations on pages 532 and 533. The approximate yield to call (AYC) is computed as follows:

$$\text{AYC} = \frac{C_i + \dfrac{P_c - P_m}{nc}}{\dfrac{P_c + P_m}{2}}$$

where

AYC = the approximate yield to call (YTC)
P_c = the call price of the bond (generally equal to par value plus one year's interest)
P_m = the market price of the bond
C_i = the annual coupon payment of bond i
nc = the number of years to first call date

[4] For a discussion of the crossover point, see Homer and Leibowitz, *Inside the Yield Book*, Chapter 4.

[5] Extensive literature exists on the refunding of bond issues, including W. M. Boyce and A. J. Kalotay, "Optimum Bond Calling and Refunding," *Interfaces* (November 1979): 36–49; R. S. Harris, "The Refunding of Discounted Debt: An Adjusted Present Value Analysis," *Financial Management* 9, no. 4 (Winter 1980): 7–12; A. J. Kalotay, "On the Structure and Valuation of Debt Refundings," *Financial Management* 11, no. 1 (Spring 1982): 41–42; and John D. Finnerty, "Evaluating the Economics of Refunding High-Coupon Sinking-Fund Debt," *Financial Management* 12, no. 1 (Spring 1983): 5–10.

This equation is comparable to APY, except that P_c has replaced P_p in the equation and nc has replaced n.

To find the AYC of a 12 percent, 20-year bond that is trading at 115 ($1,150) with 5 years remaining to first call and a call price of 112 ($1,120), we substitute these values into the above equation.

$$\text{AYC} = \frac{120 + \dfrac{1120 - 1150}{5}}{\dfrac{1120 + 1150}{2}} = 10.04\%.$$

This bond's approximate YTC is 10.04 percent, assuming that the issue will be called after 5 years at the call price of 112. To confirm that yield to call is the more conservative and more accurate value for a bond callable in 5 years, you can compute the approximate promised YTM. Using the equation on page 533 indicates a promised YTM of 10.47 percent.

To compute the YTC by the present value method, we would adjust the semiannual present value equation to give

$$P_m = \sum_{t=1}^{2nc} \frac{C_i/2}{(1 + i/2)^t} + \frac{P_c}{(1 + i/2)^{2nc}}$$

where

P_m = the current market price of the bond
C_i = the annual coupon payment of bond i
nc = the number of years to first call date
P_c = the call price of the bond

Following the present value method, we solve for i, which typically requires several computations or extrapolation to get the exact yield.

Realized (Horizon) Yield

The final measure of bond yield, **realized yield** or **horizon yield,** measures the expected rate of return of a bond that you expect to sell prior to its maturity. In terms of the equation, the investor has a holding period (*hp*) or investment horizon that is less than n. Realized (horizon) yield can be used to estimate rates of return attainable from various trading strategies. Although it is a very useful measure, it requires several additional estimates not required by the other yield measures. Specifically, the investor must estimate the expected future selling price of the bond at the end of the holding period. Also, this measure requires a specific estimate of the reinvestment rate for the coupon flows prior to the liquidation of the bond. This technique also can be used by investors to measure their actual yields after selling bonds.

COMPUTING REALIZED (HORIZON) YIELD The realized yields are variations on the promised yield equations. The approximate realized yield (ARY) is calculated as follows:

$$\text{ARY} = \frac{C_i + \dfrac{P_f - P_m}{hp}}{\dfrac{P_f + P_m}{2}}$$

NWN/502
Summers/8

where

ARY = the approximate realized (horizon) yield
C_i = the annual coupon payment of the bond i
P_f = the future selling price of the bond
P_m = the current market price of the bond
hp = the holding period of the bond (in years)

Again, the same two variables change: the holding period (hp) replaces n, and P_f replaces P_p. Keep in mind that P_f is not a contractual value but is *calculated* by defining the years remaining to maturity as $n - hp$ and by estimating a future market interest rate, i. We describe the computation of the future selling price (P_f) in the next section.

Once we determine hp and P_f, we can calculate the approximate realized yield. Assume you acquired an 8 percent, 20-year bond for \$750. Over the next two years, you expect interest rates to decline. As you know, when interest rates decline, bond prices increase. Suppose you anticipate that, when interest rates decline, the bond price will rise to \$900. The approximate realized yield in this case for the two years would be:

$$\text{ARY} = \frac{80 + \frac{900 - 750}{2}}{\frac{900 + 750}{2}} = 18.79\%.$$

The estimated high realized (horizon) yield reflects your expectation of substantial capital gains in a fairly short period of time. Similarly, the substitution of P_f and hp into the present value model provides the following realized yield model:

$$P_m = \sum_{t=1}^{2hp} \frac{C_i/2}{(1 + i/2)^t} + \frac{P_f}{(1 + i/2)^{2hp}}$$

Again, this present value model requires you to solve for the i that equates the expected cash flows from coupon payments and the estimated selling price to the current market price. Because of the small number of periods in hp, the added accuracy of this measure is somewhat marginal. It has been suggested that because realized yield measures are based on an uncertain future selling price, the approximate realized (horizon) yield method is appropriate under many circumstances. In contrast, if you are going to use this technique to measure historical performance, you should use the more accurate present value model.

You will note from the present value realized yield formula in the above equation that the coupon flows are implicitly discounted at the computed realized (horizon) yield. In many cases, this is an inappropriate assumption because available market rates might be very different from the computed realized (horizon) yield. Therefore, to derive a realistic estimate of the expected realized yield, you also should estimate your expected reinvestment rate during the investment horizon.

Therefore, to complete your understanding of computing expected realized yield for alternative investment strategies, the next section considers the calculation of future bond prices. This is followed by a section on calculating a realized (horizon) return with different reinvestment rates.

CALCULATING FUTURE BOND PRICES

Dollar bond prices need to be calculated in two instances: (1) when computing realized (horizon) yield, you must determine the future selling price (P_f) of a bond if it is to be sold before maturity or first call, and (2) when issues are quoted on a promised yield basis, as

with municipals. You can easily convert a yield-based quote to a dollar price by using the equation on page 535, which does not require iteration. (You need only solve for P_m.) The coupon (C_i) is given, as is par value (P_p), and the promised YTM, which is used as the discount rate.

Consider a 10 percent, 25-year bond with a promised YTM of 12 percent. You would compute the price of this issue as

$$P_m = 100/2 \sum_{t=1}^{50} \frac{1}{\left(1 + \frac{.120}{2}\right)^t} + 1000 \frac{1}{\left(1 + \frac{.120}{2}\right)^{50}}$$
$$= 50(15.7619) + 1000(.0543)$$
$$= \$842.40.$$

In this instance, we are determining the prevailing market price of the bond based on the current market YTM. These market figures indicate the consensus of all investors regarding the value of this bond. An investor with a required rate of return on this bond that differs from the market YTM would estimate a different value for the bond.

In contrast to the current market price, you will need to compute a future price (P_f) when estimating the expected realized (horizon) yield performance of alternative bonds. Investors or portfolio managers who consistently trade bonds for capital gains need to compute expected realized yield rather than promised yield. They would compute P_f through the following variation of the realized yield equation:

$$P_f = \sum_{t=1}^{2n-2hp} \frac{C_i/2}{(1 + i/2)^t} + \frac{P_p}{(1 + i/2)^{2n-2hp}}$$

where

P_f = the future selling price of the bond
P_p = the par value of the bond
n = the number of years to maturity
hp = the holding period of the bond (in years)
C_i = the annual coupon payment of bond i
i = the expected market YTM at the end of the holding period

This equation is a version of the present value model that calculates the expected price of the bond at the end of the holding period (hp). The term $2n - 2hp$ equals the bond's remaining term to maturity at the end of the investor's holding period, that is, the number of 6-month periods remaining after the bond is sold. Therefore, the determination of P_f is based on four variables: two that are known and two that must be estimated by the investor.

Specifically, the coupon (C_i) and the par value (P_p) are given. The investor must forecast the length of the holding period, and therefore the number of years remaining to maturity at the time the bond is sold ($n - hp$). The investor also must forecast the expected market YTM at the time of sale (i). With this information, you can calculate the future price of the bond. The real difficulty (and the potential source of error) in estimating P_f lies in predicting hp and i.

Assume you bought the 10 percent, 25-year bond just discussed at \$842, giving it a promised YTM of 12 percent. Based on an analysis of the economy and the capital market, you expect this bond's market YTM to decline to 8 percent in 5 years. Therefore, you want to compute its future price (P_f) at the end of year 5 to estimate your expected rate of return, assuming you are correct in your assessment of the decline in overall market interest rates.

NWN/502
Summers/9

As noted, you estimate the holding period (5 years), which implies a remaining life of 20 years, and the market YTM of 8 percent. A semiannual model gives a future price:

$$P_f = 50 \sum_{t=1}^{40} \frac{1}{(1.04)^t} + 1000 \frac{1}{(1.04)^{40}}$$
$$= 50(19.7928) + 1000(.2083)$$
$$= 989.64 + 208.30$$
$$= \$1,197.94.$$

Based on this estimate of the selling price, you would estimate the approximate realized (horizon) yield on this investment on an annual basis as

$$\text{APY} = \frac{100 + \frac{1198 - 842}{5}}{\frac{1198 + 842}{2}}$$
$$= \frac{100 + 71.20}{1020}$$
$$= .1678$$
$$= 16.78\%$$

REALIZED (HORIZON) YIELD WITH DIFFERENTIAL REINVESTMENT RATES

The realized yield equation on page 537 is the standard present value formula with the changes in holding period and ending price. As such, it includes the implicit reinvestment rate assumption that all cash flows are reinvested at the computed *i* rate. There may be instances where such an implicit assumption is not appropriate, given your expectations for future interest rates. Assume that current market interest rates are very high and you invest in a long-term bond (e.g., a 20-year, 14 percent coupon) to take advantage of an expected decline in rates from 14 percent to 10 percent over a 2-year period. Computing the future price (equal to \$1,330.95) and using the realized yield equation to estimate the realized (horizon) yield, we will get the following fairly high realized rate of return:

$$P_m = \$1,000$$
$$hp = 2 \text{ years}$$
$$P_f = \sum_{t=1}^{36} 70/(1 + .05)^t + \$1,000/(1.05)^{36}$$
$$= \$1,158.30 + \$172.65$$
$$= \$1,330.95$$
$$\$1,000 = \sum_{t=1}^{4} \frac{70}{(1 + i/2)^t} + \frac{1330.95}{(1 + i/2)^4}$$
$$i = 27.5\%.$$

As noted, this calculation assumes that all cash flows are reinvested at the computed *i* (27.5 percent). However, it is unlikely that during a period when market rates are going from 14 percent to 10 percent you could reinvest the coupon flows at 27.5 percent. It is more appropriate and realistic to explicitly estimate the reinvestment rates and calculate the realized yields based on your *ending-wealth position*. This procedure is more precise and realistic, and it is easier because it does not require iteration.

The basic technique calculates the value of all cash flows at the end of the holding period, which is the investor's ending-wealth value. We compare this value to our *begin-ning-wealth value* to determine the *compound rate of return that equalizes these two values*. Adding to our prior example, assume we have the following cash flows:

P_m = \$1,000
i = interest payments of \$70 in 6, 12, 18, and 24 months
P_f = \$1,330.95 (the ending market value of the bond).

The ending value of the four interest payments is determined by our assumptions regarding specific reinvestment rates. Assume each payment is reinvested at a different declining rate that holds for its time period (i.e., the first three interest payments are reinvested at progressively lower rates and the fourth interest payment is received at the end of the holding period).

i_1 at 13% for 18 months	= \$70 × (1 + .065)^3	=	\$ 84.55
i_2 at 12% for 12 months	= \$70 × (1 + .06)^2	=	78.65
i_3 at 11% for 6 months	= \$70 × (1 + .055)	=	73.85
i_4 not reinvested	= \$70 × (1.0)	=	70.00
Future Value of Interest Payments		=	\$307.05

Therefore, our total ending-wealth value is

$$\$1,330.95 + \$307.05 = \$1,638.00.$$

The compound realized (horizon) rate of return is calculated by comparing our ending-wealth value (\$1,638) to our beginning-wealth value (\$1,000) and determining what interest rate would equalize these two values over a 2-year holding period. To find this, compute the ratio of ending wealth to beginning wealth (1.638). Find this ratio in a compound value table for 4 periods (assuming semiannual compounding). Table A.3 at the end of the book indicates that the realized rate is somewhere between 12 percent (1.5735) and 14 percent (1.6890). Interpolation gives an estimated semiannual rate of 13.16 percent, which indicates an annual rate of 26.32 percent. Using a calculator or computer it is equal to $(1.638)^{1/4} - 1$. This compares to an estimate of 27.5 percent when we assume an implicit reinvestment rate of 27.5 percent.

This realized (horizon) yield computation specifically states the expected reinvestment rates as contrasted to assuming the reinvestment rate is equal to the computed realized yield. The actual assumption regarding the reinvestment rate can be very important.

A summary of the steps to calculate an expected realized (horizon) yield is as follows:

1. Calculate the future value at the horizon date of all coupon payments reinvested at estimated rates.
2. Calculate the expected sales price of the bond at your expected horizon date based on your estimate of the required yield to maturity at that time.
3. Sum the values in (1) and (2) to arrive at the total ending-wealth value.
4. Calculate the ratio of the ending wealth value to the beginning value (the purchase price of the bond). Given this ratio and the time horizon, compute the compound rate of interest that will grow to this ratio over this time horizon.

$$\left(\frac{\text{Ending-wealth value}}{\text{Beginning value}}\right)^{1/2n} - 1$$

5. If all calculations assume semiannual compounding, double the interest rate derived from (4).

NWN/502
Summers/10

PRICE AND YIELD DETERMINATION ON NONINTEREST DATES

So far, we have assumed that the investor buys (or sells) a bond precisely on the date the interest is due, so the measures are accurate only when the issues are traded on coupon payment dates. If the approximate yield method is used, sufficient accuracy normally is obtained by extrapolating for transactions on noninterest payment dates. You already are dealing with an approximation, and a bit more is probably acceptable.

However, when the semiannual model is used, and when more accuracy is necessary, another version of the price and yield model must be used for transactions on noninterest payment dates. Fortunately, the basic models need be extended only one more step because the value of an issue that trades X years, Y months, and so many days from maturity is found by extrapolating the bond value (price or yield) for the month before and the month after the day of transaction. Thus, the valuation process involves full months to maturity rather than years or semiannual periods.[6]

Having computed a value for the bond at a noninterest payment date, it is also necessary to consider the notion of *accrued interest*. Because the interest payment on a bond, which is paid every six months, is a contractual promise by the issuer, the bond investor has the right to receive a portion of the semiannual interest payment if he/she held the bond for some part of the six-month period. For example, assume an 8 percent, $1,000 par value bond that pays $40 every six months. If you sold the bond two months after the prior interest payment, you have held it for one-third of the six-month period and would have the right to one-third of the $40 ($13.33). This is referred to as the accrued interest on the bond. Therefore, when you sell the bond, there will be a calculation of the bond's remaining value until maturity, i.e., its price. What you receive is this price *plus* the accrued interest ($13.33).

YIELD ADJUSTMENTS FOR TAX-EXEMPT BONDS

Municipal bonds, Treasury issues, and many agency obligations possess one common characteristic: Their interest income is partially or fully tax-exempt. This tax-exempt status affects the valuation of taxable versus nontaxable bonds. Although you could adjust each present value equation for the tax effects, it is not necessary for our purposes. We can envision the approximate impact of such an adjustment, however, by computing the fully taxable equivalent yield, which is one of the most often cited measures of performance for municipal bonds.

The **fully taxable equivalent yield (FTEY)** adjusts the promised yield computation for the bond's tax-exempt status. To compute the FTEY, we determine the promised yield on a tax-exempt bond using one of the yield formulas and then adjust the computed yield to reflect the rate of return that must be earned on a fully taxable issue. It is measured as

$$\text{FTEY} = \frac{i}{1 - T}$$

where

i = the promised yield on the tax-exempt bond
T = the amount and type of tax exemption. (i.e., the investor's marginal tax rate)

For example, if the promised yield on the tax-exempt bond is 6 percent and the investor's marginal tax rate is 30 percent, the taxable equivalent yield would be:

[6]For a detailed discussion of these calculations, see Chapter 4 in Frank J. Fabozzi and T. Dessa Fabozzi, eds., *The Handbook of Fixed-Income Securities*, 4th ed. (Burr Ridge, IL: Irwin Professional Publishing, 1995).

$$\text{FTEY} = \frac{.06}{1 - .30} = \frac{.06}{.70} = .0857$$
$$= 8.57\%$$

The FTEY equation has some limitations. It is applicable only to par bonds or current coupon obligations such as new issues because the measure considers only interest income, ignoring capital gains, which are not tax-exempt. Therefore, we cannot use it for issues trading at a significant variation from par value (premium or discount).

BOND YIELD BOOKS

Bond value tables, commonly known as *bond books* or *yield books*, can eliminate most of the calculations for bond valuation. A bond yield table is like a present value interest factor table in that it provides a matrix of bond prices for a stated coupon rate, various terms to maturity (on the horizontal axis), and promised yields (on the vertical axis). Such a table allows you to determine either the promised yield or the price of a bond.

As might be expected, access to sophisticated calculators or computers has substantially reduced the need for and use of yield books. In addition, to truly understand the meaning of alternative yield measures, you must master the present value model and its variations that generate values for promised YTM, promised YTC, realized (horizon) yield, and bond prices.

BOND VALUATION USING SPOT RATES

Thus far, we have used the valuation model, which assumes that we discount all cash flows by one common yield, reflecting the overall required rate of return for the bond. Similarly, we compute the yield on the bond (YTM, YTC, horizon yield) as the single interest rate that would discount all the flows from the bond to equal the current market price of the bond. It was noted in the YTM calculations that this was a "promised" yield that depended on two assumptions: holding the bond to maturity and reinvesting all cash flows at the computed YTM (the IRR assumption). Notably, this second assumption often is very unrealistic because it requires a flat, constant yield curve. We know that it is extremely rare for the yield curve to be flat, much less remain constant for any period of time. The yield curve typically is upward sloping for several reasons, which we discuss in a later section. Investors at any point in time require *a different rate of return for flows at different times*. For example, if investors are buying alternative zero-coupon bonds (promising a single cash flow at maturity), they will almost always require different rates of return if they are offered a bond that matures in two years, five years, or ten years.

As mentioned earlier, the rates used to discount a flow at a point in time are called spot rates. It is possible to demonstrate the desire for different rates by examining the rates on government discount notes with different maturities (i.e., spot rates) as of early 1996 as shown in Table 16.2. These rates indicate that investors require 5.72 percent for a two-year flow, 6.08 percent for the cash flow in five years, and 6.50 percent for the cash flow in ten years. Although these differences in required rates for alternative maturities are noticeable, they are not nearly as large as they were during 1993–1994. The difference in yield between the one-year bond (5.41 percent) and the 30-year bond (6.50 percent) (referred to as the *maturity spread*) was 109 basis points in early 1996; however, it was over 250 basis points in mid-1993.

Because of these differences in spot rates across maturities, bond analysts and bond portfolio managers recognize that it is inappropriate to discount all the flows for a bond at one single rate where the rate used is often based on the yield to maturity for a government bond with that maturity. For example, when asked about the value of a particular 20-year

TABLE 16.2 YIELDS ON U.S. TREASURY STRIPS WITH ALTERNATIVE MATURITIES

Maturity	Yield
1 Year	5.41
2 Years	5.72
3 Years	5.86
4 Years	6.01
5 Years	6.08
6 Years	6.16
7 Years	6.32
8 Years	6.40
9 Years	6.50
10 Years	6.60
12 Years	6.74
14 Years	6.87
16 Years	6.95
18 Years	7.02
20 Years	7.06
25 Years	7.06
30 Years	6.50

Source: *The Wall Street Journal*, March 15, 1996.

bond rated AA, a bond trader typically will respond that the bond should trade a certain number of basis points higher than comparable maturity Treasury bonds (e.g., "plus 70 basis points"). This means that if 20-year Treasury bonds are currently yielding 7.06 percent, this bond should trade at about a 7.76 percent yield. Notably, this rate would determine the price for the bond with no consideration given to the specific cash flows of this security (i.e., high or low coupon). Therefore, there is a growing awareness that the valuation formula should be specified such that all cash flows should be discounted at spot rates consistent with the timing of the flows as follows:

$$P_m = \sum_{t=1}^{2n} \frac{C_t}{(1 + i_t/2)^t}$$

where

P_m = the market price for the bond
C_t = the cash flow at time t
n = the number of years
i_t = the spot rate for Treasury securities at time t.

Note that this valuation model requires a different discount rate for each flow so it is not possible to use the annuity concept. Also, the principal payment at the end of the year n is no different from the interest coupon flow.

To demonstrate the effect of this procedure, consider the following hypothetical spot-rate curve for the next five years (in Table 16.3) and three example bonds with equal maturities of five years, but with very different cash flows.

Beyond the differences in value because of the differences in cash flows and the rising spot-rate curve, a significant comparison is the value that would be derived using a single

TABLE 16.3 DEMONSTRATION OF DIFFERENT VALUATION OF ALTERNATIVE FIVE-YEAR MATURITY BONDS WITH UNIQUE CASH FLOWS, DISCOUNTED USING THE SPOT RATE CURVE

			Cash Flows					
			Bond A		Bond B		Bond C	
Maturity (Years)	Spot Rate	Discount Factor	$	PV	$	PV	$	PV
0.5	5.00	0.9756	60	$ 58.536	30	$ 29.268	—	—
1.0	5.20	0.9499	60	56.994	30	28.497	—	—
1.5	5.50	0.9218	60	55.308	30	27.654	—	—
2.0	5.70	0.8937	60	53.622	30	26.811	—	—
2.5	5.80	0.8668	60	52.008	30	26.004	—	—
3.0	5.90	0.8399	60	50.394	30	25.197	—	—
3.5	6.10	0.8103	60	48.618	30	24.309	—	—
4.0	6.30	0.7803	60	46.818	30	23.409	—	—
4.5	6.40	0.7532	60	45.192	30	22.596	—	—
5.0	6.50	0.7263	1,060	769.878	1,030	748.089	1,000	726.300
Total Present Value				$1,237.368		$981.834		$726.300

discount rate based on the five-year maturity of all three bonds. If we assume yield to maturity of 6 percent and 6.5 percent for five-year bonds, the values for the three bonds are:

	6%			6.5%		
Bond A	$ 60 × 8.5302	=	$ 511.81	$ 60 × 8.6350	=	$ 518.10
	$1,000 × .7441	=	744.10	$1,000 × 727.00	=	727.00
	Total Value	=	$1,255.91		=	$1,245.10
Bond B	$ 30 × 8.5302	=	$ 255.90	$ 30 × 8.6350	=	$ 259.05
	$1,000 × .7441	=	744.10	$1,000 × .7270	=	727.00
	Total Value	=	$1,000.00		=	$ 986.05
Bond C	$1,000 × .7441	=	$ 744.10	$1,000 × .7270	=	$ 727.00
	Total Value	=	$ 744.10		=	$ 727.00

Because there is a rising spot-yield curve, we know the YTM would be somewhere between these two values. The point is, valuing the bonds with either of these single rates generates a value that is greater than that derived from the spot-rate curve. This implies that the single-rate valuation technique would overvalue these bonds relative to the more appropriate technique that considers each flow as a single bond discounted by its own spot rate.

WHAT DETERMINES INTEREST RATES?

Now that we have learned to calculate various yields on bonds and to determine the value of bonds using yields and spot rates, the question arises as to what causes differences and changes in yields over time. Market interest rates cause these effects because the interest rates reported in the media are simply the prevailing YTMs for the bonds being discussed. For example, when you hear that the interest rate on long-term government bonds declined from 8.40 percent to 8.32 percent, this means that the price of this particular bond increased such that the computed YTM at the former price was 8.40 percent, but the computed YTM

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at the new, higher price is 8.32 percent. Yields and interest rates are the same. They are different terms for the same concept.

We have discussed the inverse relationship between bond prices and interest rates. When interest rates decline, the prices of bonds increase; when interest rates rise, there is a decline in bond prices. It is natural to ask which of these is the driving force—bond prices or bond interest rates? It is a simultaneous change, and you can envision either factor causing it. Most practitioners probably envision the changes in interest rates as causes because they constantly use interest rates to describe changes. They use interest rates because they are comparable across bonds, whereas the price of a bond depends not only on the interest rate, but also on its specific characteristics including its coupon and maturity. The point is, as demonstrated in Table 16.1 and Figure 16.1, when you change the interest rate (yield) on a bond, you simultaneously change its price in the opposite direction. Later in the chapter we will have a further discussion of the specific price-yield relationship for individual bonds and demonstrate that this price-yield relationship differs among bonds based on their particular coupon and maturity.

Understanding interest rates and what makes them change is necessary for an investor who hopes to maximize returns from investing in bonds. Therefore, in this section we review our prior discussion of the following topics: what causes overall market interest rates to rise and fall, why alternative bonds have different interest rates, and why the difference in rates (i.e., the yield spread) between alternative bonds changes over time. To accomplish this, we begin with a general discussion of what influences interest rates and then consider the **term structure of interest rates** (shown by yield curves), which relates the interest rates on a set of comparable bonds to their terms to maturity. The term structure is important because it implies a set of spot rates that can be used in the valuation of bonds and it reflects what investors expect to happen to interest rates in the future; it also dictates their current risk attitude. In this section, we specifically consider the calculation of spot rates and forward rates from the reported yield curve. Finally, we turn to the concept of *yield spreads*, which measure the differences in yields between alternative bonds. We describe various yield spreads and explore changes in them over time.

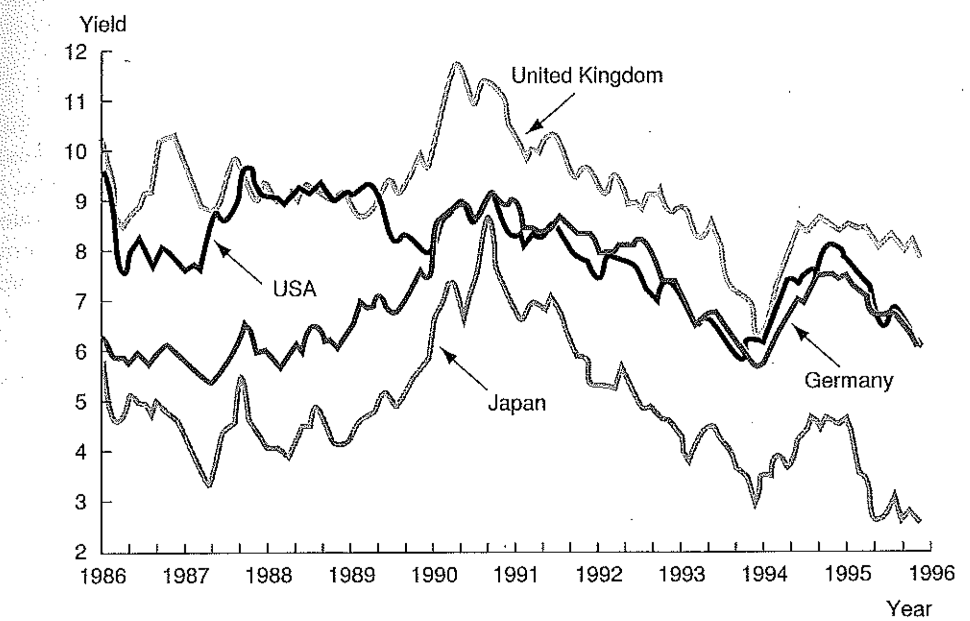
Forecasting Interest Rates

As discussed, the ability to forecast interest rates and changes in these rates is critical to successful bond investing. Later, we consider the major determinants of interest rates, but for now you should keep in mind that interest rates *are the price for loanable funds*. Like any price, they are determined by the supply and demand for these funds. On the one side, investors are willing to provide funds (the supply) at prices based on their required rates of return for a particular borrower. On the other side, borrowers need funds (the demand) to support budget deficits (government), to invest in capital projects (corporations), or to acquire durable goods (cars, appliances) or homes (individuals).

Although lenders and borrowers have some fundamental factors that determine supply and demand curves, the prices for these funds (interest rates) also are affected for short time periods by events that shift the curves. Examples include major government bond issues that affect demand, or significant changes in Federal Reserve monetary policy that affect the supply of money.

Our treatment of interest rate forecasting recognizes that you must be aware of the basic determinants of interest rates and monitor these factors. We also recognize that detailed forecasting of interest rates is a very complex task that is best left to professional economists. Therefore, our goal as bond investors and bond portfolio managers is to monitor current and expected interest rate behavior. We should attempt to continuously assess the major factors that affect interest rate behavior but also rely on others—such as economic

FIGURE 16.3 **YIELDS OF INTERNATIONAL LONG-TERM GOVERNMENT BONDS: MONTHLY 1986–1995**



Note: For Japan and Germany, rates on newly issued securities are given until October 1986; secondary market yields are quoted thereafter.

Sources: Federal Reserve Bank, Telerate.

consulting firms, banks, or investment banking firms—for detailed insights on such topics as the real RFR and the expected rate of inflation.[7] This is precisely the way most bond portfolio managers operate.

Fundamental Determinants of Interest Rates

As shown in Figure 16.3, average interest rates for long-term (10-year) U.S. government bonds during the period from late 1990 through 1995 went from 9 percent to 6 percent. These results were midway between the United Kingdom and Japan. UK bonds went from about 12 percent to 8 percent, while the rate on Japanese government bonds declined from 8.7 percent to about 2.7 percent. As a bond investor, you should understand *why* these differences exist and *why* interest rates changed.

As you know from your knowledge of bond pricing, bond prices increased dramatically during periods when market interest rates dropped, and some bond investors experienced very attractive returns. In contrast, some investors experienced substantial losses during periods when interest rates increased. A casual analysis of this chart, which covers about 9 years, indicates the need for monitoring interest rates. Essentially, the factors causing interest rates (i) to rise or fall are described by the following model:

$$i = \text{RFR} + I + \text{RP}$$

[7] Sources of information on the bond market and interest rate forecasts would include Merrill-Lynch's *Fixed Income Weekly* and *World Bond Market Monitor*; Goldman, Sach's *Financial Market Perspectives*; Kidder, Peabody's *Economic Outlook and Chartbook*; and the Federal Reserve Bank of St. Louis, *Monetary Trends*.

where

RFR = the real risk-free rate of interest
I = the expected rate of inflation
RP = the risk premium.

The relationship shown in this equation should be familiar from our presentations in Chapters 1 and 13. It is a simple but complete statement of interest rate behavior. The more difficult task is estimating the *future* behavior of such variables as real growth, expected inflation, and economic uncertainty. In this regard, interest rates, like stock prices, are extremely difficult to forecast with any degree of accuracy.[8] Alternatively, we can visualize the source of changes in interest rates in terms of the economic conditions and issue characteristics that determine the rate of return on a bond:

$$\begin{aligned} i &= f(\text{Economic Forces} + \text{Issue Characteristics}) \\ &= (\text{RFR} + I) + \text{RP}. \end{aligned}$$

This rearranged version of the previous equation helps isolate the determinants of interest rates.[9]

EFFECT OF ECONOMIC FACTORS The real risk-free rate of interest (RFR) is the economic cost of money, that is, the opportunity cost necessary to compensate individuals for forgoing consumption. As discussed previously, it is determined by the real growth rate of the economy with short-run effects due to ease or tightness in the capital market.

The expected rate of inflation is the other economic influence on interest rates. We add the expected level of inflation (I) to the real risk-free rate (RFR) to specify the nominal RFR, which is a market rate like the current rate on government T-bills. Given the stability of the real RFR, it is clear that the wide swings in nominal risk-free interest rates during the years covered by Figure 16.3 occurred because of expected inflation.[10] Besides the unique country and exchange rate risk that we discuss in the section on risk premiums, differences in the rates of inflation between countries have a major impact on their level of interest rates.

To sum up, one way to estimate the nominal RFR is to begin with the real growth rate of the economy, adjust for short-run ease or tightness in the capital market, and then adjust this real rate of interest for the expected rate of inflation.

Another approach to estimating the nominal rate or changes in the rate is the macroeconomic view, where the supply and demand for loanable funds are the fundamental economic determinants of i. As the supply of loanable funds increases, the level of interest rates declines, other things being equal. Several factors influence the supply of funds. Government monetary policies imposed by the Federal Reserve have a significant impact on the supply of money. The savings patterns of U.S. and non-U.S. investors also affect the

[8] For an overview of interest rate forecasting, see Frank J. Jones and Benjamin Wolkowitz, "The Determinants of Interest Rates," and W. David Woolford, "Forecasting Interest Rates," in *The Handbook of Fixed-Income Securities*, 4th ed., edited by Frank J. Fabozzi and T. Dessa Fabozzi (Burr Ridge, IL: Irwin Professional Publishing, 1995).

[9] For an extensive exploration of interest rates and interest rate behavior, see James C. Van Horne, *Financial Market Rates and Flows*, 4th ed. (Englewood Cliffs, NJ: Prentice-Hall, 1993).

[10] In this regard, see R. W. Hafer, "Inflation: Assessing Its Recent Behavior and Future Prospects," Federal Reserve Bank of St. Louis *Review* 65, no. 7 (August–September 1983): 36–41; and C. Alan Garner, "How Useful Are Leading Indicators of Inflation?" Federal Reserve Bank of Kansas City *Economic Review* 80, no. 2 (Second Quarter 1995): 5–18.

supply of funds. Non-U.S. investors have become a stronger influence on the U.S. supply of loanable funds during recent years, as shown by the significant purchases of U.S. securities by non-U.S. investors, most notably the Japanese prior to a pullback in 1992. It is widely acknowledged that this foreign addition to the supply of funds has been very beneficial to the United States since it has helped reduce interest rates and cost of capital.

Interest rates increase when the demand for loanable funds increases. The demand for loanable funds is affected by the capital and operating needs of the U.S. government, federal agencies, state and local governments, corporations, institutions, and individuals. Federal budget deficits increase the Treasury's demand for loanable funds. Likewise, the level of consumer demand for funds to buy houses, autos, and appliances affects rates, as does corporate demand for funds to pursue investment opportunities. The total of all groups determines the aggregate demand and supply of loanable funds and the level of the nominal RFR.[11]

THE IMPACT OF BOND CHARACTERISTICS The interest rate of a specific bond issue is influenced not only by all the factors that affect the nominal RFR, but also by its unique issue characteristics. These issue characteristics influence the bond's risk premium (RP). The economic forces that determine the nominal RFR affect all securities, whereas issue characteristics are unique to individual securities, market sectors, or countries. Thus, the differences in the yields of corporate and Treasury bonds are not caused by economic forces, but rather by different issue characteristics that cause differences in the risk premiums.

Bond investors separate the risk premium into four components:

1. The quality of the issue as determined by its risk of default relative to other bonds
2. The term to maturity of the issue, which can affect yield and price volatility
3. Indenture provisions, including collateral, call features, and sinking-fund provisions
4. Foreign bond risk, including exchange rate risk and country risk

Of the four factors, quality and maturity have the greatest impact on the risk premium for domestic bonds, while exchange rate risk and country risk are important components of risk for non-U.S. bonds.

The credit quality of a bond reflects the ability of the issuer to service outstanding debt obligations. This information is largely captured in the ratings issued by the bond rating firms. As a result, bonds with different ratings have different yields. For example, AAA-rated obligations possess lower risk of default than BBB obligations, so they can provide lower yield.

Notably, the risk premium differences between bonds of different quality levels have changed dramatically over time, depending on prevailing economic conditions. When the economy experiences a recession or a period of economic uncertainty, the desire for quality increases, and investors bid up prices of higher-rated bonds, which reduces their yields. This difference in yield is referred to as the quality spread. It also has been suggested by Dialynas and Edington that this yield spread is influenced by the volatility of interest rates.[12] This variability in the risk premium over time was demonstrated and discussed in Chapters 1 and 13.

[11] For an example of an estimate of the supply and demand for funds in the economy, see *Prospects for Financial Markets in 1996* (New York: Salomon Bros., 1995). This is an annual publication of Salomon Brothers that gives an estimate of the flow of funds in the economy and discusses its effect on various currencies and interest rates, making recommendations for portfolio strategy on the basis of these expectations.

[12] Chris P. Dialynas and David H. Edington, "Bond Yield Spreads: A Postmodern View," *Journal of Portfolio Management* 19, no. 1 (Fall 1992): 68–75.

Term to maturity also influences the risk premium because it affects an investor's level of uncertainty as well as the price volatility of the bond. In the section on the term structure of interest rates, we will discuss the typical positive relationship between the term to maturity of a bond issue and its interest rate.

As discussed in Chapter 15, indenture provisions indicate the collateral pledged for a bond, its callability, and its sinking-fund provisions. Collateral gives protection to the investor if the issuer defaults on the bond because the investor has a specific claim on some assets in case of liquidation.

Call features indicate when an issuer can buy back the bond prior to its maturity. A bond is called by an issuer when interest rates have declined so typically it is not to the advantage of the investor who must reinvest the proceeds at a lower interest rate. Obviously, an investor will charge the issuer for including the call option, and the cost of the option (which is a higher yield) will increase with the level of interest rates. Therefore, more protection against having the bond called reduces the risk premium. The significance of call protection increases during periods of high interest rates. When you buy a bond with a high coupon, you want protection from having it called away when rates decline.[13]

A sinking fund reduces the investor's risk and causes a lower yield for several reasons. First, a sinking fund reduces default risk because it requires the issuer to reduce the outstanding issue systematically. Second, purchases of the bond by the issuer to satisfy sinking-fund requirements provide price support for the bond because of the added demand. These purchases by the issuer also contribute to a more liquid secondary market for the bond because of the increased trading. Finally, sinking-fund provisions require that the issuer retire a bond before its stated maturity, which causes a reduction in the issue's average maturity. The decline in average maturity tends to reduce the risk premium of the bond much as a shorter maturity would reduce yield.[14]

We know that foreign currency exchange rates change over time and that this increases the risk of global investing. Differences in the variability of exchange rates among countries arise because the trade balances and rates of inflation differ among countries. More volatile trade balances and inflation rates in a country make its exchange rates more volatile, which will add to the uncertainty of future exchange rates. These factors increase the exchange rate risk premium.

In addition to the ongoing changes in exchange rates, investors always are concerned with the political and economic stability of a country. If investors are unsure about the political environment or the economic system in a country, they will increase the risk premium they require to reflect this country risk.[15]

TERM STRUCTURE OF INTEREST RATES

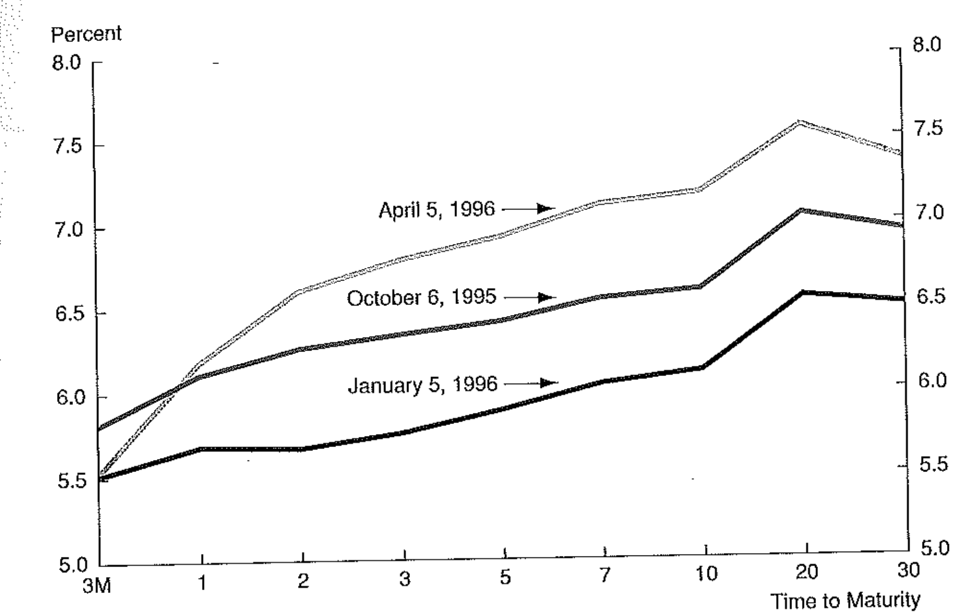
The term structure of interest rates (or the *yield curve,* as it is more popularly known) is a static function that relates the term to maturity to the yield to maturity for a sample of

[13] William Marshall and Jess B. Yawitz, "Optimal Terms of the Call Provision on a Corporate Bond," *Journal of Financial Research* 3, no. 3 (Fall 1980): 203–211; and Michael G. Ferri, "Systematic Return Risk and the Call Risk of Corporate Debt Instruments," *Journal of Financial Research* 1, no. 1 (Winter 1978): 1–13.

[14] For a further discussion of sinking funds, see Edward A. Dyl and Michael D. Joehnk, "Sinking Funds and the Cost of Corporate Debt," *Journal of Finance* 34, no. 4 (September 1979): 887–893; A. J. Kalotay, "On the Management of Sinking Funds," *Financial Management* 10, no. 2 (Summer 1981): 34–40; and A. J. Kalotay, "Sinking Funds and the Realized Cost of Debt," *Financial Management* 11, no. 1 (Spring 1982): 43–54.

[15] In this regard, see Martin Fridson, "Sovereign Risk from a Corporate Bond Analyst Perspective," and Brian Murphy, David Won, and Deepak Gulrajani, "Valuation and Risk Analysis of International Bonds." Both are in *The Handbook of Fixed-Income Securities,* 4th ed., edited by Frank J. Fabozzi and T. Dessa Fabozzi (Burr Ridge, IL: Irwin Professional Publishing, 1995).

FIGURE 16.4 **TREASURY YIELD CURVES**



Source: Curves by authors using data from *Federal Reserve Bulletin* (Washington, D.C., various issues.)

bonds at *a given point in time.*[16] Thus, it represents a cross section of yields for a category of bonds that are comparable in all respects but maturity. Specifically, the quality of the issues should be constant, and ideally you should have issues with similar coupons and call features within a single industry category. You can construct different yield curves for Treasuries, government agencies, prime-grade municipals, AAA utilities, and so on. The accuracy of the yield curve will depend on the comparability of the bonds in the sample.

As an example, Figure 16.4 shows yield curves for a sample of U.S. Treasury obligations. It is based on the yield to maturity information for a set of comparable Treasury issues from a publication such as the *Federal Reserve Bulletin* or *The Wall Street Journal.* These promised yields were plotted on the graph, and a yield curve was drawn that represents the general configuration of rates. These data represent yield curves at three different points in time to demonstrate the changes in yield levels and in the shape of the yield curve over time.

All yield curves, of course, do not have the same shape as those in Figure 16.4. Although individual yield curves are static, their behavior over time is quite fluid. As shown, the level of the curve decreased from October, 1995 to January, 1996 and then the slope increased to April 1, 1996. Also, the shape of the yield curve can undergo dramatic alterations, following one of the four patterns shown in Figure 16.5. The rising yield curve is the most common and tends to prevail when interest rates are at low or modest levels.

[16] For a discussion of the theory and empirical evidence, see Richard W. McEnally and James V. Jordan, "The Term Structure of Interest Rates," in *The Handbook of Fixed-Income Securities,* 4th ed., edited by Frank J. Fabozzi and T. Dessa Fabozzi (Burr Ridge, IL: Irwin Professional Publishing, 1995).

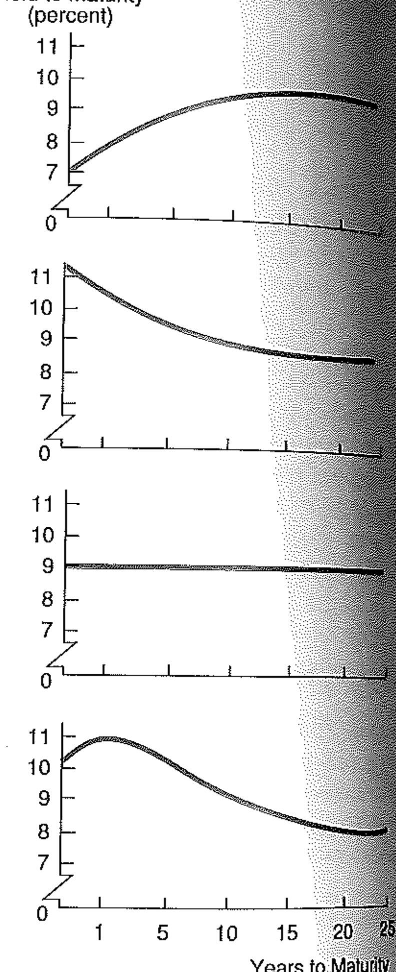
FIGURE 16.5 **TYPES OF YIELD CURVES**

A Rising Yield Curve is formed when the yields on short-term issues are low and rise consistently with longer maturities and flatten out at the extremes.

A Declining Yield Curve is formed when the yields on short-term issues are high and yields on subsequently longer maturities decline consistently.

A Flat Yield Curve has approximately equal yields on short-term and long-term issues.

A Humped Yield Curve is formed when yields on intermediate-term issues are above those on short-term issues and the rates on long-term issues decline to levels below those for the short-term and then level out.



The declining yield curve tends to occur when rates are relatively high. The flat yield curve rarely exists for any period of time. The humped yield curve prevails when extremely high rates are expected to decline to more normal levels. Note that the slope of the curve tends to level off after 15 years.

Why does the term structure assume different shapes? Three major theories attempt to explain this: the expectations hypothesis, the liquidity preference hypothesis, and the segmented market hypothesis.

Before we discuss these three alternative hypotheses, we must first discuss two previously noted rates that not only are an integral part of the term structure, but important in the valuation of bonds. The next two subsections will deal with the specification and computation of *spot rates* and *forward rates*. Earlier, we discussed and used spot rates to value bonds with the idea that any coupon bond can be viewed as a collection of zero coupon securities.

TABLE 16.4 **MATURITY AND YIELD TO MATURITY FOR HYPOTHETICAL TREASURY SECURITIES**

Maturity (Years)	Coupon Rate	Price	Yield-to-Maturity
0.50	0.0000	96.15	0.0800
1.00	0.0000	92.19	0.0830
1.50	0.0850	99.45	0.0890
2.00	0.0900	99.64	0.0920
2.50	0.1100	103.49	0.0940
3.00	0.0950	99.49	0.0970

Source: *Federal Reserve Bulletin*, Moody's *Bond Guide*.

CREATING THE THEORETICAL SPOT-RATE CURVE[17] Earlier in the chapter, we discussed the notion that the yield on a zero coupon bond for a given maturity is the spot rate for the maturity. Specifically, the spot rate is defined as the discount rate for a cash flow at a specific maturity. At that time, we used the rates on a series of zero coupon government bonds created by stripping coupon government bonds.

In this case, we will construct a theoretical spot-rate curve from the observable yield curve that is based on the existing yields of Treasury bills and the most recent Treasury coupon securities (referred to as *on-the-run* Treasury issues). One might expect the theoretical spot-rate curve and the spot-rate curve derived from the stripped zero-coupon bonds used earlier to be the same. The fact is, while they are close, they will not be exactly the same because the stripped zero-coupon bonds will not be as liquid as the on-the-run issues. In addition, there are instances where institutions will have a strong desire for a particular spot maturity and this preference will distort the term structure relationship. Therefore, while it is possible to use the stripped zero-coupon curve for a general indication, if you are going to use the spot rates for significant valuation, you would want to use the theoretical spot-rate curve.

The process of creating a theoretical spot-rate curve from coupon securities is called *bootstrapping* wherein it is assumed that the value of the Treasury coupon security should be equal to the value of the package of zero-coupon securities that duplicates the coupon bond's cash flow. Table 16.4 lists the maturity and YTM for six hypothetical Treasury bonds that will be used to calculate the initial spot rates.

Consider the six-month Treasury bill in Table 16.4. As discussed earlier, a Treasury bill is a zero-coupon instrument so its annualized yield of 8 percent is equal to the spot rate. Similarly, for the one-year Treasury, the cited yield of 8.3 percent is equal to the one-year spot rate. Given these two spot rates, we can compute the spot rate for a theoretical 1.5-year zero-coupon Treasury. The price should equal the present value of three cash flows from an actual 1.5-year coupon Treasury, where the yield used for discounting is the spot rate corresponding to the cash flow.

Using $100 as par, *the cash flow* for the 1.5-year coupon Treasury is as follows:

0.5 years	.085 × $100 × .5	=	$ 4.25
1.0 years	.085 × $100 × .5	=	$ 4.25
1.5 years	.085 × $100 × .5 + $100	=	$104.25

[17]This discussion of the theoretical spot-rate cuve and the subsequent presentation on calculating forward rates draws heavily from Frank J. Fabozzi, "The Structure of Interest Rates," in *The Handbook of Fixed Income Securities* 4th ed., edited by Frank J. Fabozzi and T. Dessa Fabozzi (Burr Ridge, IL: Irwin Professional Publishing, 1995).

The present value of the cash flows discounted as the appropriate spot rates is then

$$\frac{4.25}{(1+z_1)^1}+\frac{4.25}{(1+z_2)^2}+\frac{104.25}{(1+z_3)^3}$$

where

z_1 = One-half the annualized six-month theoretical spot rate
z_2 = One-half the one-year theoretical spot rate
z_3 = One-half the 1.5-year theoretical spot rate

Because the six-month spot rate and one-year spot rate are 8.0 percent and 8.3 percent, respectively, we know that

$$z_1 = .04 \quad and \quad z_2 = .0415.$$

We can compute the present value of the 1.5-year coupon Treasury security as

$$\frac{4.25}{(1.0400)^1}+\frac{4.25}{(1.0415)^2}+\frac{104.25}{(1+z_3)^3}$$

Because the price of the 1.5-year coupon Treasury security (from Table 16.4) is $99.45, the following relationship must hold:

$$99.45=\frac{4.25}{(1.0400)^1}+\frac{4.25}{(1.0415)^2}+\frac{104.25}{(1+z_3)^3}$$

We can solve for the theoretical 1.5 year spot rate as follows:

$$99.45 = 4.08654 + 3.91805 + \frac{104.25}{(1+z_3)^3}$$

$$91.44541 = \frac{104.25}{(1+z_3)^3}$$

$$\frac{104.25}{91.44541} = (1+z_3)^3$$

$$(1+z_3)^3 = 1.140024$$

$$z_3 = .04465$$

Doubling this yield, we obtain the bond-equivalent yield of .0893 or 8.93 percent, which is the theoretical 1.5-year spot rate. That rate is the rate that the market would apply to a 1.5-year zero-coupon Treasury, if such a security existed.

Given the theoretical 1.5-year spot rate, we can obtain the theoretical two-year spot rate. The cash flow for the two-year coupon Treasury in Table 16.4 is

0.5 years	.090 × $100 × .5	=	$ 4.50
1.0 years	.090 × $100 × .5	=	$ 4.50
1.5 years	.090 × $100 × .5	=	$ 4.50
2.0 years	.090 × $100 × .5 + 100	=	$104.50

The present value of the cash flow is then

$$\frac{4.50}{(1+z_1)^1}+\frac{4.50}{(1+z_2)^2}+\frac{4.50}{(1+z_3)^3}+\frac{104.50}{(1+z_4)^4}$$

where

z_4 = One-half the two-year theoretical spot rate

Because the six-month spot rate, one-year spot rate, and 1.5-year spot rate are 8 percent, 8.3 percent, and 8.93 percent respectively, then

$$z_1 = .04 \quad z_2 = .0415 \quad and \quad z_3 = .04465$$

Therefore, the present value of the two-year coupon Treasury security is

$$\frac{4.50}{(1.0400)^1}+\frac{4.50}{(1.0415)^2}+\frac{4.50}{(1.04465)^3}+\frac{104.50}{(1+z_4)^4}$$

Because the price of the two-year coupon Treasury security is $99.64, the following relationship must hold:

$$99.64=\frac{4.50}{(1.0400)^1}+\frac{4.50}{(1.0415)^2}+\frac{4.50}{(1.04465)^3}+\frac{104.50}{(1+z_4)^4}$$

We can solve for the theoretical two-year spot rate as follows:

$$99.64 = 4.32692 + 4.14853 + 3.94730 + \frac{104.50}{(1+z_4)^4}$$

$$87.21725 = \frac{104.50}{(1+z_4)^4}$$

$$(1+z_4)^4 = 1.198158$$

$$z_4 = .046235$$

Doubling this yield, we obtain the theoretical two-year spot rate bond-equivalent yield of 9.247 percent.

One can follow this approach sequentially to derive the theoretical 2.5-year spot rate from the calculated vales of z_1, z_2, z_3, z_4 (the six-month, one-year, 1.5-year, and two-year rates), and the price and the coupon of the bond with a maturity of 2.5 years. Further, one could derive the theoretical spot rate for three years. The spot rates thus obtained are shown in Table 16.5. They represent the term structure of interest rates for maturities up to three years, based upon the prevailing bond price quotations.

As shown, with a rising YTM curve, the theoretical spot rate will increase at a faster rate such that the difference increases with maturity (i.e., the theoretical spot-rate curve will be above a positively sloped YTM curve).

TABLE 16.5 THEORETICAL SPOT RATES

Maturity (Years)	Yield-to-Maturity	Theoretical Spot Rate
0.50	0.0800	0.08000
1.00	0.0830	0.08300
1.50	0.0890	0.08930
2.00	0.0920	0.09247
2.50	0.0940	0.09468
3.00	0.0970	0.09787

CALCULATING FORWARD RATES FROM THE SPOT-RATE CURVE

Now that we have derived the theoretical spot-rate curve, it is possible to determine what this curve implies regarding the market's expectation of *future* short-term rates, which are referred to as *forward rates*. The following illustrates the process of extrapolating the information about expected future interest rates.

Consider an investor who has a one-year investment horizon and is faced with the following two alternatives:

Alternative 1: Buy a one-year Treasury bill.

Alternative 2: Buy a six-month Treasury bill and when it matures in six months, buy another six-month Treasury bill.

The investor will be indifferent between the two alternatives if they produce the same return on the one-year investment horizon. The investor knows the spot rate on the six-month Treasury bill and the one-year Treasury bill. However, she does not know what yield will be available on a six-month Treasury bill six months from now. The yield on a six-month Treasury bill six months from now is called a **forward rate.** Given the spot rate for the six-month Treasury bill and the one-year bill, we can determine the forward rate on a six-month Treasury bill *that will make the investor indifferent between the two alternatives.*

At this point, however, we need to digress briefly and recall several present value and investment relationships. First, if you invested in a one-year Treasury bill, you would receive $100 at the end of one year. The price of the one-year Treasury bill would be

$$\frac{100}{(1 + z_2)^2}$$

where

z_2 is one-half the bond-equivalent yield of the theoretical one-year spot rate

Second, suppose you purchased a six-month Treasury bill for $X. At the end of six months, the value of this investment would be

$$X(1 + z_1)$$

where

z_1 is one-half the bond-equivalent yield of the theoretical six-month spot rate

Let ${}_{t+.5}r_{.5}$ represent one-half the forward rate (expressed as a bond-equivalent yield) on a six-month Treasury bill (.5) available six months from now (t + .5). If the investor were to

renew her investment by purchasing that bill at that time, then the future dollars available at the end of the year from the $X investment would be

$$X(1 + z_1)\,(1 + {}_{t+.5}r_{.5}) = 100$$

Third, it is easy to use that formula to find out how many dollars the investor must invest in order to get $100 one year from now. This can be found as follows:

$$(1 + z_1)\,(1 + {}_{t+.5}r_{.5}) = 100$$

which gives us

$$X = \frac{100}{(1 + z_1)\,(1 + {}_{t+.5}r_{.5})}$$

We are now prepared to return to the investor's choices and analyze what that situation says about forward rates. The investor will be indifferent between the two alternatives confronting her if she makes the same dollar investment and receives $100 from both alternatives at the end of one year. That is, the investor will be indifferent if

$$\frac{100}{(1 + z_2)^2} = \frac{100}{(1 + z_1)\,(1 + {}_{t+.5}r_{.5})}$$

Solving for ${}_{t+.5}r_{.5}$ we get

$${}_{t+.5}r_{.5} = \frac{(1 + z_2)^2}{(1 + z_1)} - 1$$

Doubling r gives the bond-equivalent yield for the six-month forward rate six months from now.

We can illustrate the use of this formula with the theoretical spot rates shown in Table 16.5. From that table, we know that

Six-month bill spot rate = .080 so z_1 = .0400
One-year bill spot rate = .083 so z_2 = .0415

Substituting into the formula, we have

$${}_{t+.5}r_{.5} = \frac{(1.0415)^2}{1.0400} - 1$$
$$= .043$$

Therefore, the forward rate six months from now (t + .5) on a six-month Treasury security, quoted annually, is 8.6 percent (.043 × 2). Let us confirm our results. The price of a one-year Treasury bill with $100 maturity is

$$\frac{100}{(1.0415)^2} = 92.19$$

If $92.19 is invested for six months at the six-month spot rate of 8 percent, the amount at the end of six months would be

$$92.19(1.0400) = 95.8776$$

If $95.8776 is reinvested for another six months in a six-month Treasury bill offering 4.3 percent for six months (8.6 percent annually), the amount at the end of one year would be

$$95.8876(1.043) = 100$$

Both alternatives will have the same $100 payoff if the six-month Treasury bill yield six months from now is 4.3 percent (8.6 percent on a bond-equivalent basis). This means that if an investor is guaranteed a 4.3 percent yield on a six-month Treasury bill six months from now, she will be indifferent between the two alternatives.

We used the theoretical spot rates to compute the forward rate. The resulting forward rate is called the *implied forward rate*.

It is possible to use the yield curve to calculate the implied forward rate for any time in the future for any investment horizon. This would include six-month or one-year forward rates for each year in the future. The one-year forward rates would be designated as follows:

$_{t+1}r_1$ = **the one-year forward rate, one year from now (t + 1)**
$_{t+2}r_1$ = **the one-year forward rate, two years from now (t + 2)**
$_{t+3}r_1$ = **the one-year forward rate, three years from now (t + 3)**

Given the calculations, it is clear that with a rising spot-rate curve, the forward-rate curve would be above the spot-rate curve. From Table 16.5, we have the following one-year spot rates, which imply the following one-year forward rates:

Maturity (Years)	Spot Rates	One-Year Forward Rates
1.0	.08300	
2.0	.09247	.1020
3.0	.09787	.1087

Therefore:

$$_{t+1}r_1 = \frac{(1.09247)^2}{(1.08300)} - 1 = \frac{1.19349}{1.08300} - 1 = .1020$$

$$_{t+2}r_1 = \frac{(1.09787)^3}{(1.09247)^2} - 1 = \frac{1.32328}{1.19349} - 1 = .1087$$

TERM-STRUCTURE THEORIES

EXPECTATIONS HYPOTHESIS According to the expectations hypothesis, the shape of the yield curve results from the interest rate expectations of market participants. More specifically, it holds that *any long-term interest rate simply represents the geometric mean of current and future 1-year interest rates expected to prevail over the maturity of the issue.* In essence, the term structure involves a series of intermediate and long-term interest rates, each of which is a reflection of the geometric average of current and expected 1-year interest rates. Under such conditions, the equilibrium long-term rate is the rate the long-term bond investor would expect to earn through successive investments in short-term bonds over the term to maturity of the long-term bond.

Generally, this relationship can be formalized as follows:

$$(1 + {}_tR_n) = [(1 + {}_tR_1)(1 + {}_{t+1}r_1) \ldots (1 + {}_{t+n-1}r_1)]^{1/N}$$

where

R_n = **the actual long-term rate**
N = **the term to maturity (in years) of long issue**
R = **the current 1-year rate**
$_{t+1}r_t$ = **the expected 1-year yield during some future period, $t + i$ (these future 1-year rates are referred to as *forward rates*).**

Given the relationship set forth in this equation, the formula for computing the one-period forward rate beginning at time $t + n$ and implied in the term structure at time t is:

$$1 + {}_{t+n}r_{1t} = \frac{(1 + {}_tR_{1t})\,(1 + {}_{t+1}r_{1t})\,(1 + {}_{t+2}r_{1t}) \ldots (1 + {}_{t+n-1}r_{1t})\,(1 + {}_{t+n}r_{1t})}{(1 + {}_tR_{1t})\,(1 + {}_{t+1}r_{1t}) \ldots (1 + {}_{t+n-1}r_{1t})}$$

$$= \frac{(1 + {}_tR_{n+1})^{n+1}}{(1 + {}_tR_n)^n}$$

$$_{t+n}r_{1t} = \frac{(1 + {}_tR_{n+1})^{n+1}}{(1 + {}_tR_n)^n} - 1$$

where $_{t+n}r_{1t}$ is the 1-year forward rate prevailing at $t + n$, using the term structure at time t.

Assume that the 5-year spot rate is 10 percent (${}_tR_5 = .10$) and the 4-year spot rate is 9 percent (${}_tR_4 = .09$). The forward 1-year rate 4 years from now implied by these spot rates can be calculated as follows:

$$_{t+4}r_{1t} = \frac{(1 + {}_tR_5)^5}{(1 + {}_tR_4)^4} - 1$$

$$= \frac{(1 + .10)^5}{(1 + .09)^4} - 1$$

$$= \frac{1.6105}{1.4116} - 1$$

$$= 1.1409 - 1 = .1409 = 14.09\%$$

The term structure at time t implies that the 1-year spot rate 4 years from now (during Year 5) will be 14.09 percent. This concept and formula can be used to derive future rates for multiple years. Thus, the 2-year spot rate that will prevail 3 years from now could be calculated using the 3-year spot rate and the 5-year spot rate. The general formula for computing the j-period forward rate beginning at time $t + n$ as of time t is

$$_{t+n}r_{jt} = \sqrt[j]{\frac{(1 + {}_tR_{n+j})^{n+j}}{(1 + {}_tR_n)^n}} - 1$$

As a practical approximation of the equation at the top of this page, it is possible to use the *arithmetic* average of 1-year rates to generate long-term yields.

The expectations theory can explain any shape of yield curve. Expectations for rising short-term rates in the future cause a rising yield curve; expectations for falling short-term rates in the future will cause long-term rates to lie below current short-term rates, and the yield curve will decline. Similar explanations account for flat and humped yield curves.

Consider the following explanation by the expectations hypothesis of the shape of the term structure of interest rates using arithmetic averages:

$_1R_1 = 5\frac{1}{2}\%$ the 1-year rate of interest prevailing now (period t)
$_{t+1}r_1 = 6\%$ the 1-year rate of interest expected to prevail next year (period $t + 1$)
$_{t+2}r_1 = 7\frac{1}{2}\%$ the 1-year rate of interest expected to prevail 2 years from now (period $t + 2$)
$_{t+3}r_1 = 8\frac{1}{2}\%$ the 1-year rate of interest expected to prevail 3 years from now (period $t + 3$)

Using these values, and the known rate on a 1-year bond, we compute rates on 2-, 3-, or 4-year bonds (designated R_2, R_3, and R_4) as follows:

$_1R_1 = 5\frac{1}{2}$ percent
$_1R_2 = (0.055 + 0.06)/2 = 5.75$ percent
$_1R_3 = (0.055 + 0.06 + 0.075)/3 = 6.33$ percent
$_1R_4 = (0.055 + 0.06 + 0.075 + 0.085)/4 = 6.88$ percent

In this illustration (which uses the arithmetic average as an approximation of the geometric mean), the yield curve is upward-sloping because, at present, investors expect future short-term rates to be above current short-term rates. This is not the formal method for constructing the yield curve. Rather, it is constructed on the basis of the prevailing promised yields for bonds with different maturities.

The expectations hypothesis attempts to explain *why* the yield curve is upward-sloping, downward-sloping, humped, or flat by explaining the expectations implicit in yield curves with different shapes. The evidence is fairly substantial and convincing that the expectations hypothesis is a workable explanation of the term structure. Because of the supporting evidence, its relative simplicity, and the intuitive appeal of the theory, the expectations hypothesis of the term structure of interest rates is rather widely accepted.

Besides the theory and empirical support, it is also possible to present a scenario wherein investor actions will cause the yield curve postulated by the theory. The expectations hypothesis predicts a declining yield curve when interest rates are expected to fall in the future rather than rise. In such a case, long-term bonds would be considered attractive investments because investors would want to lock in prevailing higher yields (which are not expected to be as high in the future) or they would want to capture the increase in bond prices (as capital gains) that will accompany a decline in rates. By the same reasoning, investors will avoid short-term bonds or sell them and reinvest the funds in long-term bonds. The point is, investor expectations will reinforce the declining shape of the yield curve as they bid up the prices of long-maturity bonds (forcing yields to decline) and short-term bond issues are avoided or sold (so prices decline and yields rise). At the same time, there is confirming action by suppliers of bonds. Specifically, government or corporate issuers will avoid selling long bonds at the current high rates, waiting until the rates decline. In the meantime, they will issue short-term bonds, if needed, while waiting for lower rates. Therefore, in the long-term market you will have an increase in demand and a decline in the supply and vice versa in the short-term market. These shifts between long- and short-term maturities will continue until equilibrium occurs or expectations change.

LIQUIDITY PREFERENCE HYPOTHESIS The theory of liquidity preference holds that long-term securities should provide higher returns than short-term obligations because investors are willing to sacrifice some yields to invest in short-maturity obligations to avoid the higher price volatility of long-maturity bonds. Another way to interpret the liquidity preference hypothesis is to say that lenders prefer short-term loans, and, to induce them to lend long term, it is necessary to offer higher yields.

The liquidity preference theory contends that uncertainty causes investors to favor short-term issues over bonds with longer maturities because short-term bonds can easily be converted into predictable amounts of cash should unforeseen events occur. This theory argues that the yield curve should slope upward and that any other shape should be viewed as a temporary aberration.

This theory can be considered an extension of the expectations hypothesis because the formal liquidity preference position contends that the liquidity premium inherent in the yields for longer maturity bonds should be added to the expected future rate in arriving at long-term yields. Specifically, the liquidity premium (L) compensates the investor in long-term bonds for the added uncertainty because of less stable prices. Because the liquidity premium (L) is provided to compensate the long-term investor, it is simply a variation of the equation on page 557 as follows:

$$(1 + {}_tR_N) = [(1 + {}_tR_1)\,(1 + {}_{t+1}r_1 + L_2) \ldots (1 + {}_{t+N-1}r_1 + L_n]^{1/N}$$

In this specification, the Ls are not the same, but would be expected to increase with time. The liquidity preference theory has been found to possess some strong empirical support.[18]

To see how the liquidity preference theory predicts future yields and how it compares with the pure expectations hypothesis, let us predict future long-term rates from a single set of 1-year rates: 6 percent, 7.5 percent, and 8.5 percent. The liquidity preference theory suggests that investors add increasing liquidity premiums to successive rates to derive actual market rates. As an example, they might arrive at rates of 6.3 percent, 7.9 percent, and 9.0 percent.

As a matter of historical fact, the yield curve shows a definite upward bias, which implies that some combination of the expectations theory and the liquidity preference theory will more accurately explain the shape of the yield curve than either of them alone. Specifically, actual long-term rates consistently tend to be above what is envisioned from the price expectations hypothesis, which implies the existence of a liquidity premium.

SEGMENTED MARKET HYPOTHESIS Despite meager empirical support, a third theory for the shape of the yield curve is the segmented market hypothesis, which enjoys wide acceptance among market practitioners. Also known as the *preferred habitat*, the *institutional theory*, or the *hedging pressure theory*, it asserts that different institutional investors have different maturity needs that lead them to confine their security selections to specific maturity segments. That is, investors supposedly focus on short-, intermediate-, or long-term securities. This theory contends that the shape of the yield curve ultimately is a function of these investment policies of major financial institutions.

Financial institutions tend to structure their investment policies in line with factors such as their tax liabilities, the types and maturity structure of their liabilities, and the level of earnings demanded by depositors. For example, because commercial banks are subject to normal corporate tax rates and their liabilities are generally short- to intermediate-term time and demand deposits, they consistently invest in short- to intermediate-term municipal bonds.

[18] See Reuben A. Kessel, "The Cyclical Behavior of the Term Structure of Interest Rates," Occasional Paper 91, National Bureau of Economic Research, 1965; Phillip Cagan, *Essays on Interest Rates* (New York: Columbia University Press for the National Bureau of Economic Research, 1969); and J. Huston McCulloch, "An Estimate of the Liquidity Premium," *Journal of Political Economy* 83, no. 1 (January–February 1975): 95–119.

The segmented market theory contends that the business environment, along with legal and regulatory limitations, tends to direct each type of financial institution to allocate its resources to particular types of bonds with specific maturity characteristics. In its strongest form, the segmented market theory holds that the maturity preferences of investors and borrowers are so strong that investors never purchase securities outside their preferred maturity range to take advantage of yield differentials. As a result, the short- and long-maturity portions of the bond market are effectively segmented, and yields for a segment depend on the supply and demand *within* that maturity segment.

TRADING IMPLICATIONS OF THE TERM STRUCTURE Information on maturities can help you formulate yield expectations by simply observing the shape of the yield curve. If the yield curve is declining sharply, historical evidence suggests that interest rates will probably decline. Expectations theorists would suggest that you need to examine only the prevailing yield curve to predict the direction of interest rates in the future.

Based on these theories, bond investors use the prevailing yield curve to predict the shapes of future yield curves. Using this prediction and knowledge of current interest rates, investors can determine expected yield volatility by maturity sector. In turn, the maturity segments that experience the greatest yield changes give the investor the largest potential price appreciation.[19]

Yield Spreads

Another technique that helps make good bond investments or profitable trades is the analysis of *yield spreads*—the differences in promised yields between bond issues or segments of the market at any point in time. Such differences are specific to the particular issues or segments of the bond market. Thus they add to the rates determined by the basic economic forces (RFR + *I*).

There are four major yield spreads:

1. Different *segments* of the bond market may have different yields. For example, pure government bonds will have lower yields than government agency bonds, and government bonds have much lower yields than corporate bonds.
2. Bonds in different *sectors* of the same market segment may have different yields. For example, prime-grade municipal bonds will have lower yields than good-grade municipal bonds; you will find spreads between AA utilities and BBB utilities, or between AAA industrial bonds and AAA public utility bonds.
3. Different *coupons* or *seasoning* within a given market segment or sector may cause yield spreads. Examples include current coupon government bonds versus deep-discount governments or recently issued AA industrials versus seasoned AA industrials.
4. Different *maturities* within a given market segment or sector also cause differences in yields. You will see yield spreads between short-term agency issues and long-term agency issues, or between 3-year prime municipals and 25-year prime municipals.

The differences among these bonds cause yield spreads that may be either positive or negative. More important, *the magnitude or the direction of a spread can change over time*. These changes in size or direction of yield spreads offer profit opportunities. We say that the spread narrows whenever the differences in yield become smaller; it widens as the differences increase. Table 16.6 contains data on a variety of past yield spreads.

[19] Gikas A. Hourdouvelis, "The Predictive Power of the Term Structure During Recent Monetary Regimes," *Journal of Finance* 43, no. 2 (June 1988): 339–356.

TABLE 16.6 **SELECTED MEAN YIELD SPREADS (REPORTED IN BASIS POINTS)**

Comparisons	1984	1985	1986	1987	1988	1989	1990	1991	1992
1. Short Governments—Long Governments[a]	+10	+111	+108	+96	+72	+3	+48	+127	+210
2. Long Governments—Long Aaa Corporates[b]	+72	+62	+88	+74	+73	+68	+58	+61	+62
3. Long Municipals—Long Aaa Corporates[c]	+272	+226	+170	+175	+203	+203	+220	+199	+185
4. Long Aaa Municipals—Long Baa Municipals[d]	+77	+98	+81	+103	+47	+40	+104	+103	+84
5. AA Utilities—BBB Utilities[e]	+88	+90	+70	+76	+74	+42	+41	+46	+31
6. AA Utilities—AA Industrials[e]	−51	−11	+33	+19	−65	−20	−20	−9	−18

[a] Median yield to maturity of a varying number of bonds with 2 to 5 years' maturity and more than 10 years, respectively.
[b] Long Aaa corporates based on yields to maturity on selected long-term bonds.
[c] Long-term municipal issues based on Bond Buyer Series, a representative list of high-quality municipal bonds with a 20-year period to maturity being maintained.
[d] General obligation municipal bonds only.
[e] Based on a changing list of representative issues.

Source: *Federal Reserve Bulletin*, Moody's *Bond Guide*.

As a bond investor, you should evaluate yield spread changes because these changes influence bond price behavior and comparative return performance. You should attempt to identify (1) any normal yield spread that is expected to become abnormally wide or narrow in response to an anticipated swing in market interest rates, or (2) an abnormally wide or narrow yield spread that is expected to become normal.

Economic and market analysis help develop these expectations of potential for yield spreads to change. Taking advantage of these changes requires a knowledge of historical spreads and an ability to *predict* not only future total market changes, but also why and when specific spreads will change.[20]

WHAT DETERMINES THE PRICE VOLATILITY FOR BONDS?

In this chapter, we have learned about alternative bond yields, how to calculate them, what determines bond yields (interest rates), and what causes them to change. Now that we understand why yields change, we can logically ask: what is the effect of these yield changes on the prices and rates of return for different bonds? We have discussed the inverse relationship between changes in yields and the price of bonds, so we can now discuss *the specific factors that affect the amount of price change for a yield change* in different bonds. This can also be referred to as the *interest rate sensitivity* of a bond. This section lists the specific factors that affect bond price changes for a given change in interest rates (i.e., the interest rate sensitivity of a bond) and demonstrates the effect for different bonds.

A given change in interest rates can cause vastly different percentage price changes for alternative bonds, which implies different interest rate sensitivity. This section will help

[20] An article that identifies four determinants of relative market spreads and suggests scenarios when they will change is Chris P. Dialynas and David H. Edington, "Bond Yield Spreads: A Postmodern View," *Journal of Portfolio Management* 19, no. 1 (Fall 1992): 68–75.

you understand what causes these differences in interest rate sensitivity. To maximize your rate of return from your knowledge of a decline in interest rates, for example, you need to know which bonds will benefit the most from the yield change. This section helps you make this bond selection decision.

Throughout this section, we talk about bond price changes or bond price volatility interchangeably. A bond price change is measured as the percentage change in the price of the bond, computed as follows:

$$\frac{EPB}{BPB} - 1$$

where

EPB = the ending price of the bond
BPB = the beginning price of the bond.

Bond price volatility also is measured in terms of percentage changes in bond prices. A bond with high price volatility or high interest rate sensitivity is one that experiences large percentage price changes for a given change in yields.

Bond price volatility is influenced by more than yield behavior alone. Malkiel used the bond valuation model to demonstrate that the market price of a bond is a function of four factors: (1) its par value, (2) its coupon, (3) the number of years to its maturity, and (4) the prevailing market interest rate.[21] Malkiel's mathematical proofs showed the following relationships between yield (interest rate) changes and bond price behavior:

1. Bond prices move inversely to bond yields (interest rates).
2. For a given change in yields (interest rates), longer-maturity bonds post larger price changes; thus, bond price volatility is *directly* related to term to maturity.
3. Price volatility (percentage of price change) increases at a diminishing rate as term to maturity increases.
4. Price movements resulting from equal absolute increases or decreases in yield are not symmetrical. A decrease in yield raises bond prices by more than an increase in yield of the same amount lowers prices.
5. Higher coupon issues show smaller percentage price fluctuation for a given change in yield; thus, bond price volatility is *inversely* related to coupon.

Homer and Leibowitz showed that the absolute level of market yields also affects bond price volatility.[22] As the level of prevailing yields rises, the price volatility of bonds increases, *assuming a constant percentage change in market yields*. It is important to note that if you assume a constant percentage change in yield, the basis-point change will be greater when rates are high. For example, a 25 percent change in interest rates when rates are at 4 percent will be 100 basis points; the same 25 percent change when rates are at 8 percent will be a 200 basis-point change. In the discussion of bond duration, we will see that this difference in basis point change is important.

Tables 16.7, 16.8, and 16.9 demonstrate these relationships assuming semiannual compounding. Table 16.7 demonstrates the effect of maturity on price volatility. In all four maturity classes, we assume a bond with an 8 percent coupon and assume that the discount rate (YTM) changes from 7 percent to 10 percent. The only difference among the four cases is the maturities of the bonds. The demonstration involves computing the value of each bond at a 7 percent yield and at a 10 percent yield and noting the percentage change in price. As shown, this change in yield caused the price of the 1-year bond to decline by only 2.9 percent; the 30-year bond declined by almost 29 percent. Clearly, the longer-maturity bond experienced the greater price volatility.

Also, price volatility increased at a decreasing rate with maturity. When maturity doubled from 10 years to 20 years, the price increased by less than 50 percent (from 18.5 percent to 25.7 percent). A similar change occurred when going from 20 years to 30 years. Therefore, this table demonstrates the first three of our price–yield relationships: Bond price is inversely related to yields, bond price volatility is positively related to term to maturity, and bond price volatility increases at a decreasing rate with maturity.

It also is possible to demonstrate the fourth relationship with this table. Using the 20-year bond, if you computed the percentage change in price related to an *increase* in rates (e.g., from 7 percent to 10 percent), you would get the answer reported—a 25.7 percent decrease. In contrast, if you computed the effect on price of a *decrease* in yields from 10 percent to 7 percent, you would get a 34.7 percent increase in price ($1,115 vs. $828). This demonstrates that prices change more in response to a decrease in rates (from 10 percent to 7 percent) than to a comparable increase in rates (from 7 percent to 10 percent).

Table 16.8 demonstrates the coupon effect. In this set of examples, all the bonds have equal maturity (20 years) and experience the same change in YTM (from 7 percent to 10 percent). The table shows the inverse relationship between coupon rate and price volatility: The smallest coupon bond (the zero) experienced the largest percentage price change (almost 45 percent), versus a 24 percent change for the 12 percent coupon bond.

[21] Burton G. Malkiel, "Expectations, Bond Prices, and the Term Structure of Interest Rates," *Quarterly Journal of Economics* 76, no. 2 (May 1962): 197–218.

[22] Sidney Homer and Martin L. Leibowitz, *Inside the Yield Book* (Englewood Cliffs, NJ: Prentice-Hall, 1972).

TABLE 16.7 **EFFECT OF MATURITY ON BOND PRICE VOLATILITY**

	Present Value of an 8 Percent Bond ($1,000 par value)							
Term to Maturity	1 Year		10 Years		20 Years		30 Years	
Discount rate (YTM)	7%	10%	7%	10%	7%	10%	7%	10%
Present value of interest	$ 75	$ 73	$ 569	$498	$ 858	$686	$1,005	$757
Present value of principal	934	907	505	377	257	142	132	54
Total value of bond	$1,009	$980	$1,074	$875	$1,115	$828	$1,137	$811
Percentage change in total value	–2.9		–18.5		–25.7		–28.7	

TABLE 16.8 **EFFECT OF COUPON ON BOND PRICE VOLATILITY**

	Present Value of 20-Year Bond ($1,000 par value)							
	0 Percent Coupon		3 Percent Coupon		8 Percent Coupon		12 Percent Coupon	
Discount rate (YTM)	7%	10%	7%	10%	7%	10%	7%	10%
Present value of interest	$ 0	$ 0	$322	$257	$ 858	$686	$1,287	$1,030
Present value of principal	257	142	257	142	257	142	257	142
Total value of bond	$257	$142	$579	$399	$1,115	$828	$1,544	$1,172
Percentage change in total value	–44.7		–31.1		–25.7		–24.1	

TABLE 16.9 **EFFECT OF YIELD LEVEL ON BOND PRICE VOLATILITY**

	Present Value of a 20-Year, 4 Percent Bond ($1,000 Par Value)							
	(1)		(2)		(3)		(4)	
	Low Yields		Intermediate Yields		High Yields		100 Basis Point Change at High Yields	
Discount rate (YTM)	3%	4%	6%	8%	9%	12%	9%	[illegible]
Present value of interest	$ 602	$ 547	$462	$396	$370	$301	$370	[illegible]
Present value of principal	562	453	307	208	175	97	175	[illegible]
Total value of bond	$1,164	$1,000	$769	$604	$545	$398	$545	[illegible]
Percentage change in total value	−14.1		−21.5		−27.0		−11.0	

Table 16.9 demonstrates the yield level effect. In these examples, all the bonds have the same 20-year maturity and the same 4 percent coupon. In the first three cases, the YTM changed by a constant 33.3 percent (i.e., from 3 percent to 4 percent, from 6 percent to 8 percent, and from 9 percent to 12 percent). Note that the first change is 100 basis points, the second is 200 basis points, and the third is 300 basis points. The results in the first three columns confirm the statement that when higher rates change by a *constant percentage*, the change in the bond price is larger when the rates are at a higher level.

The fourth column shows that if you assume a *constant basis-point change in yields*, you get the opposite results. Specifically, a 100 basis point change in yields from 3 percent to 4 percent provides a price change of 14.1 percent, while the same 100 basis point change from 9 percent to 10 percent results in a price change of only 11 percent. Therefore, the yield level effect can differ, depending on whether the yield change is a constant percentage change or a constant basis-point change.

Thus, the price volatility of a bond for a given change in yield (i.e., its interest rate sensitivity) is affected by the bond's coupon, its term to maturity, the level of yields (depending on what kind of change in yield), and the direction of the yield change. However, although both the level and direction of change in yields affect price volatility, they cannot be used for trading strategies. When yields change, the two variables that have a dramatic effect on a bond's interest rate sensitivity are coupon and maturity.

TRADING STRATEGIES Knowing that coupon and maturity are the major variables that influence a bond's interest rate sensitivity, we can develop some strategies for maximizing rates of return when interest rates change. Specifically, if you expect a major *decline* in interest rates, you know that bond prices will increase, so you want a portfolio of bonds with the *maximum interest rate sensitivity* so that you will enjoy maximum price changes (capital gains) from the change in interest rates. In this situation, the previous discussion regarding the effect of maturity and coupon indicates that you should attempt to build a portfolio of long-maturity bonds with low coupons (ideally a long-term zero coupon bond). A portfolio of such bonds should experience the maximum price appreciation for a given decline in market interest rates.

In contrast, if you expect an *increase* in market interest rates, you know that bond prices will decline, and you want a portfolio with *minimum interest rate sensitivity* to minimize the capital losses caused by the increase in rates. Therefore, you would want to change your portfolio to short-maturity bonds with high coupons. This combination should provide minimal price volatility for a change in market interest rates.

DURATION MEASURES Because the price volatility (interest rate sensitivity) of a bond varies inversely with its coupon and directly with its term to maturity, it is necessary to determine the best combination of these two variables to achieve your objective. This effort would benefit from a composite measure that considered both coupon and maturity.

A measure of the interest-rate sensitivity of a bond is referred to as **duration**. This concept and its development as a tool in bond analysis and portfolio management has existed for over 50 years. Notably, several specifications of duration have been derived. First, *Macaulay duration*, developed almost 60 years ago by Frederick Macaulay, is a measure of the time flow of cash from a bond.[23] A modified version of Macaulay duration can be used under certain conditions to indicate the price volatility of a bond in response to interest rate changes. Second, **modified duration** is derived by making a small adjustment (modification) to the Macaulay duration value. As noted above, under certain restrictive conditions (most important, there are no embedded options) modified duration can provide an approximation to the interest-rate sensitivity of a bond (or any financial asset). Finally, **effective duration** is a direct measure of the interest rate sensitivity of a bond (or any financial instrument). Because of the development of many new financial instruments, which have very unique cash flows *that change with interest rates*, effective duration has become widely used because of its flexibility and ability to provide a useful measure of interest rate sensitivity—the primary goal of duration. Therefore, in this section we discuss and demonstrate these three duration measures, including their limitations.

MACAULAY DURATION Macaulay showed that the duration of a bond was a more appropriate measure of time characteristics than the term to maturity of the bond because duration considers both the repayment of capital at maturity and the size and timing of coupon payments prior to final maturity. Using annual compounding, duration (D) is

$$D = \frac{\sum_{t=1}^{n} \frac{C_t(t)}{(1+i)^t}}{\sum_{t=1}^{n} \frac{C_t}{(1+i)^t}}$$

where

t = the time period in which the coupon or principal payment occurs
C_t = the interest or principal payment that occurs in period t
i = the yield to maturity on the bond.

The denominator in this equation is the price of a bond as determined by the present value model. The numerator is the present value of all cash flows *weighted according to the time to cash receipt*. The following example, which demonstrates the specific computations for two bonds, shows the procedure and highlights some of the properties of duration. Consider the following two sample bonds:

	Bond A	Bond B
Face Value	$1,000	$1,000
Maturity	10 years	10 years
Coupon	4%	8%

[23] Frederick R. Macaulay, *Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields, and Stock Prices in the United States Since 1856* (New York: National Bureau of Economic Research, 1938).

NWN/502
Summers/22

TABLE 16.10

COMPUTATION OF MACAULAY DURATION (ASSUMING 8 PERCENT MARKET YIELD)

BOND A

(1) Year	(2) Cash Flow	(3) PV at 8%	(4) PV of Flow	(5) PV as % of Price	(6) (1) × (5)
1	$ 40	.9259	$ 37.04	.0506	.0506
2	40	.8573	34.29	.0469	.0938
3	40	.7938	31.75	.0434	.1302
4	40	.7350	29.40	.0402	.1608
5	40	.6806	27.22	.0372	.1860
6	40	.6302	25.21	.0345	.2070
7	40	.5835	23.34	.0319	.2233
8	40	.5403	21.61	.0295	.2360
9	40	.5002	20.01	.0274	.2466
10	1,040	.4632	481.73	.6585	6.5850
Sum			$ 731.58	1.0000	8.1193
Duration = 8.12 Years					

BOND B

(1) Year	(2) Cash Flow	(3) PV at 8%	(4) PV of Flow	(5) PV as % of Price	(6) (1) × (5)
1	$ 80	.9259	$ 74.07	.0741	.0741
2	80	.8573	68.59	.0686	.1372
3	80	.7938	63.50	.0635	.1906
4	80	.7350	58.80	.0588	.1906
5	80	.6806	54.44	.0544	.2720
6	80	.6302	50.42	.0504	.3024
7	80	.5835	46.68	.0467	.3269
8	80	.5403	43.22	.0432	.3456
9	80	.5002	40.02	.0400	.3600
10	1,080	.4632	500.26	.5003	5.0030
Sum			$1,000.00	1.0000	7.2470
Duration = 7.25 Years					

Assuming annual interest payments and an 8 percent yield to maturity on the bonds, duration is computed as shown in Table 16.10. If duration is computed by discounting flows using the yield to maturity of the bond, it is called *Macaulay duration.*

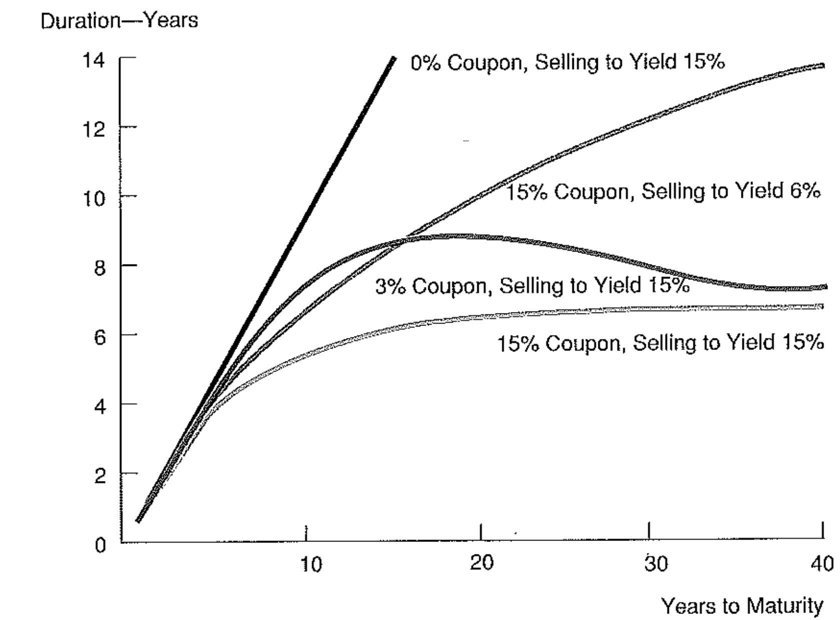
Characteristics of Macaulay duration This example illustrates several characteristics of Macaulay duration. First, the Macaulay duration of a bond with coupon payments always will be less than its term to maturity because duration gives weight to these interim payments.

Second, there is *an inverse relationship between coupon and duration.* A bond with a larger coupon will have a shorter duration because more of the total cash flows come earlier in the form of interest payments. As shown in Table 16.10, the 8 percent coupon bond has a shorter duration than the 4 percent coupon bond.

A zero coupon bond or a pure discount bond such as a Treasury bill will have *duration equal to its term to maturity.* In Table 16.10, if you assume a single payment at maturity, duration will equal term to maturity because the only cash flow comes in the final (maturity) year.

Third, there is *generally a positive relationship between term to maturity and Macaulay duration,* but duration increases at a decreasing rate with maturity. Therefore, a bond with longer term to maturity almost always will have a higher duration. The relationship is not direct because as maturity increases the present value of the principal declines in value.

FIGURE 16.6 **DURATION VS. MATURITY**



As shown in Figure 16.6, the shape of the duration-maturity curve depends on the coupon and the yield to maturity. The curve for a zero coupon bond is a straight line, indicating that duration equals term to maturity. In contrast, the curve for a low coupon bond selling at a deep discount (due to a high YTM) will turn down at long maturities, which means that under these conditions the longer-maturity bond will have lower duration.

Fourth, all else the same, there is an *inverse relationship between YTM and duration.* A higher yield to maturity of a bond reduces its duration. As an example, in Table 16.10, if the yield to maturity had been 12 percent rather than 8 percent, the duration for the 4 percent bond would have gone from 8.12 to 7.75, and the duration of the 8 percent bond would have gone from 7.25 to 6.80.[24]

Finally, sinking funds and call provisions can have a dramatic effect on a bond's duration. They can change the total cash flows for a bond and, therefore, significantly change its duration. Between these two factors, the characteristic that causes the greatest uncertainty is the call feature because it is difficult to estimate when it will be exercised since it is a function of changes in interest rates. We consider this further when we discuss the effect of the call feature on the convexity of a bond.

A summary of Macaulay duration characteristics is as follows:

- The duration of a zero coupon bond will *equal* its term to maturity.
- The duration of a coupon bond always will be less than its term to maturity.
- There is an *inverse* relationship between coupon and duration.

[24]These properties are discussed and demonstrated in Frank K. Reilly and Rupinder Sidhu, "The Many Uses of Bond Duration," *Financial Analysts Journal* 36, no. 4 (July–August 1980): 58–72; and Frank J. Fabozzi, Mark Pitts, and Ravi E. Dattatreya, "Price Volatility Characteristics of Fixed Income Securities," in *The Handbook of Fixed-Income Securities*, 4th ed., edited by Frank J. Fabozzi and T. Dessa Fabozzi (Burr Ridge, IL: Irwin Professional Publishing, 1995).

- There is generally a *positive* relationship between term to maturity and duration. Note that duration increases at a decreasing rate with maturity. Also, the duration of a deep discount bond will decline at very long maturities (over 20 years).
- There is an *inverse* relationship between yield to maturity and duration.
- Sinking funds and call provisions can cause a dramatic change in the duration of a bond. The effect of the call feature is discussed in a subsequent section.

Modified Duration and Bond Price Volatility

An adjusted measure of duration called *modified duration* can be used to approximate the interest rate sensitivity of a noncallable bond. Modified duration equals Macaulay duration (computed in Table 16.10) divided by 1 plus the current yield to maturity divided by the number of payments in a year. As an example, a bond with a Macaulay duration of 10 years, a yield to maturity (i) of 8 percent, and semiannual payments would have a modified duration of

$$D_{mod} = 10/\left(1 + \frac{.08}{2}\right)$$
$$= 10/(1.04) = 9.62$$

It has been shown, both theoretically and empirically, that price movements of option-free bonds *will vary proportionally* with modified duration for *small changes in yields*.[25] Specifically, as shown in the equation below, an estimate of the percentage change in bond price equals the change in yield times modified duration:

$$\frac{\Delta P}{P} \times 100 = -D_{mod} \times \Delta i$$

where

ΔP = the change in price for the bond
P = the beginning price for the bond
$-D_{mod}$ = the modified duration of the bond
Δi = the yield change in basis points divided by 100. For example, if interest rates go from 8.00 to 8.50 percent, $\Delta i = 50/100 = 0.50$.

Consider a bond with Macaulay $D = 8$ years and $i = 0.10$. Assume that you expect the bond's YTM to decline by 75 basis points (e.g., from 10 percent to 9.25 percent). The first step is to compute the bond's modified duration as follows:

$$D_{mod} = 8/\left(1 + \frac{.10}{2}\right)$$
$$= 8/(1.05) = 7.62$$

The estimated percentage change in the price of the bond is as follows:

$$\% \ \Delta P = -(7.62) \times \frac{-75}{100}$$
$$= (-7.62) \times (-.75)$$
$$= 5.72$$

[25] A generalized proof of this is contained in Michael H. Hopewell and George Kaufman, "Bond Price Volatility and Term to Maturity: A Generalized Respecification," *American Economic Review* 63, no. 4 (September 1973): 749–753. The importance of the specification, "for small changes in yields," will become clear when we discuss convexity in the next section. Because modified duration is an approximate measure of interest rate sensitivity, the years label is not appropriate.

TABLE 16.11 **BOND DURATION IN YEARS FOR BOND YIELDING 6 PERCENT UNDER DIFFERENT TERMS**

	Coupon Rates			
Years to Maturity	0.02	0.04	0.06	0.08
1	0.995	0.990	0.985	0.981
5	4.756	4.558	4.393	4.254
10	8.891	8.169	7.662	7.286
20	14.981	12.980	11.904	11.232
50	19.452	17.129	16.273	15.829
100	17.567	17.232	17.120	17.064
∞	17.167	17.167	17.167	17.167

Source: L. Fisher and R. L. Weil, "Coping with the Risk of Interest Rate Fluctuations: Returns to Bondholders from Naive and Optimal Strategies," *Journal of Business* 44, no. 4 (October 1971): 418. Copyright © 1971 by The University of Chicago Press. Reprinted by permission of The University of Chicago Press.

This indicates that the bond price should increase by approximately 5.72 percent in response to the 75 basis point decline in YTM. If the price of the bond before the decline in interest rates was $900, the price after the decline in interest rates should be approximately $900 × 1.0572 = $951.48.

The modified duration is always a negative value for a noncallable bond because of the inverse relationship between yield changes and bond price changes. Also, remember that this formulation provides an *estimate* or *approximation* of the percent change in the price of the bond. The following section on convexity shows that this formula that uses modified duration provides an exact estimate of the percentage price change only for very small changes in yields of option-free securities.

TRADING STRATEGIES USING MODIFIED DURATION We know that the longest duration security provides the maximum price variation. Table 16.11 demonstrates that numerous ways exist to achieve a given level of duration. The following discussion indicates that an active bond investor can use this measure of interest rate sensitivity to structure a portfolio to take advantage of changes in market yields.

If you expect a *decline* in interest rates, you should *increase* the average modified duration of your bond portfolio to experience maximum price volatility. If you expect an *increase* in interest rates, you should *reduce* the average modified duration of your portfolio to minimize your price decline. Note that the modified duration of your portfolio is the market value weighted average of the modified durations of the individual bonds in the portfolio.

Bond Convexity

Modified duration allows us to estimate bond price changes for a change in interest rates. However, the equation we used to make this calculation (on page 568) is accurate only for *very small changes* in market yields. We will see that the accuracy of the estimate of the price change deteriorates with larger changes in yields because the modified duration calculation is a *linear* approximation of a bond price change that follows a *curvilinear* (convex) function. To understand the effect of this **convexity**, we must consider the price–yield relationship for alternative bonds.[26]

[26] For a further discussion of this topic, see Mark L. Dunetz and James M. Mahoney, "Using Duration and Convexity in the Analysis of Callable Bonds," *Financial Analysts Journal* 44, no. 3 (May–June 1988): 53–73.

NWN/502
Summers/24

TABLE 16.12 **PRICE–YIELD RELATIONSHIPS FOR ALTERNATIVE BONDS**

A. 12 Percent, 20-Year		B. 12 Percent, 3-Year		C. Zero Coupon, 30-Year	
Yield	Price	Yield	Price	Yield	Price
1.0%	$2,989.47	1.0%	$1,324.30	1.0%	[illegible]
2.0	2,641.73	2.0	1,289.77	2.0	[illegible]
3.0	2,346.21	3.0	1,256.37	3.0	[illegible]
4.0	2,094.22	4.0	1,224.06	4.0	[illegible]
5.0	1,878.60	5.0	1,192.78	5.0	[illegible]
6.0	1,693.44	6.0	1,162.52	6.0	[illegible]
7.0	1,533.88	7.0	1,133.21	7.0	[illegible]
8.0	1,395.86	8.0	1,104.84	8.0	[illegible]
9.0	1,276.02	9.0	1,077.37	9.0	[illegible]
10.0	1,171.59	10.0	1,050.76	10.0	[illegible]
11.0	1,080.23	11.0	1,024.98	11.0	[illegible]
12.0	1,000.00	12.0	1,000.00	12.0	[illegible]

THE PRICE–YIELD RELATIONSHIP FOR BONDS Because the price of a bond is the present value of its cash flows at a particular discount rate, if you are given the coupon, maturity, and a yield for a bond, you can calculate its price at a point in time. The price-yield curve provides a set of prices for a specific maturity-coupon bond at a point in time using a range of yields to maturity (discount rates). As an example, Table 16.12 lists the computed prices for a 12 percent, 20-year bond assuming yields from 1 percent to 12 percent. The table shows that if you discount the flows from this bond at a yield of 1 percent, you would get a price of $2,989.47; discounting these same flows at 10 percent gives a price of $1,171.59. The graph of these prices relative to the yields that produced them in Figure 16.7 indicates that the price–yield relationship for this bond is not a straight line but a curvilinear relationship. That is, it is convex.

Two points are important about the price–yield relationship:

1. This relationship can be applied to a single bond, a portfolio of bonds, or any stream of future cash flows.
2. The convex price–yield relationship will differ among bonds or other streams, depending on the nature of the cash flow stream, that is, its coupon and maturity. For example, the price–yield relationship for a high-coupon, short-term security will be almost a straight line because the price does not change as much for a change in yields (e.g., the 12 percent, 3-year bond in Table 16.12). In contrast, the price–yield relationship for a low-coupon, long-term bond will curve radically (i.e., be very convex), as shown by the zero coupon, 30-year bond in Table 16.12. These differences in convexity are shown graphically in Figure 16.8. The curved nature of the price–yield relationship is referred to as the bond's *convexity*.

As shown by the graph in Figure 16.8, because of the convexity of the relationship, as yield increases, the rate at which the price of the bond declines becomes slower. Similarly, when yields decline, the rate at which the price of the bond increases becomes faster. Therefore, convexity is considered a desirable trait.

FIGURE 16.7 **PRICE–YIELD RELATIONSHIP AND MODIFIED DURATION AT 4 PERCENT YIELD**

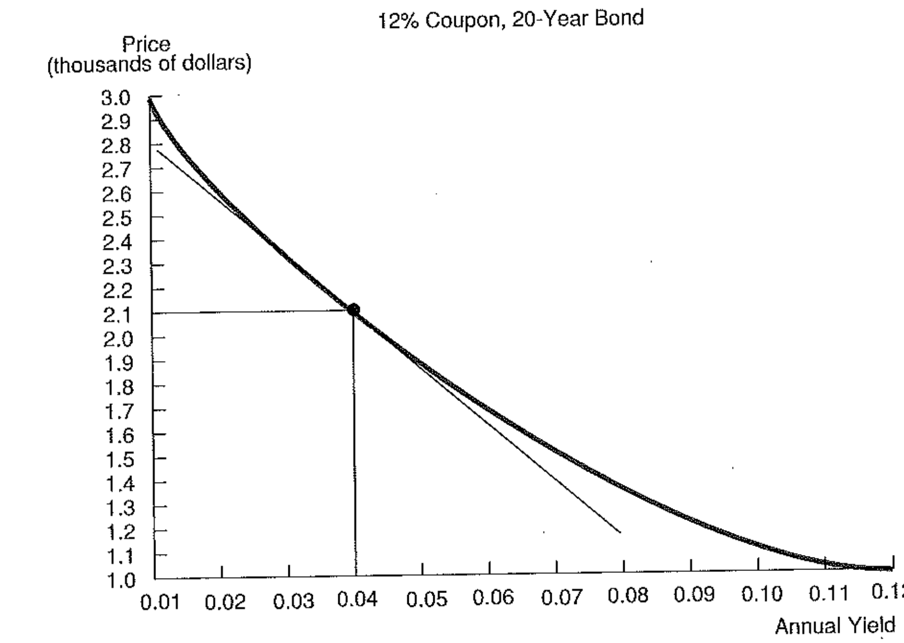


FIGURE 16.8 **PRICE–YIELD CURVES FOR ALTERNATIVE BONDS**

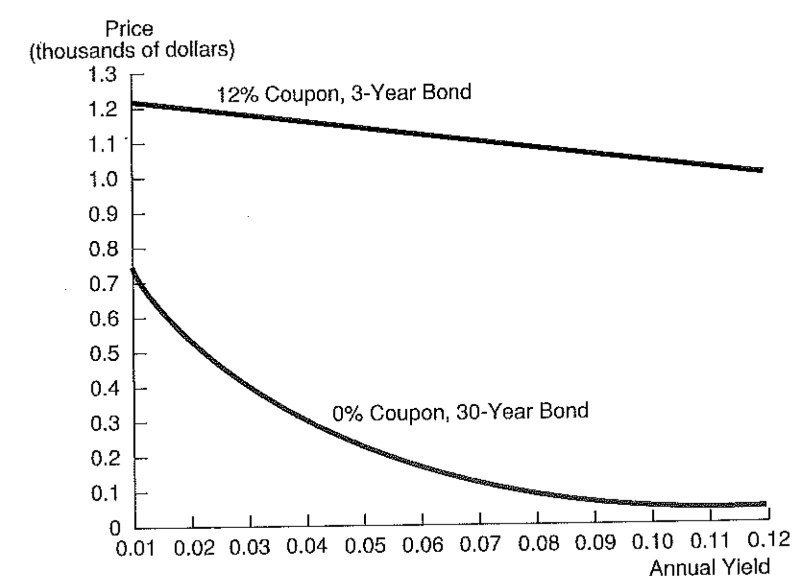
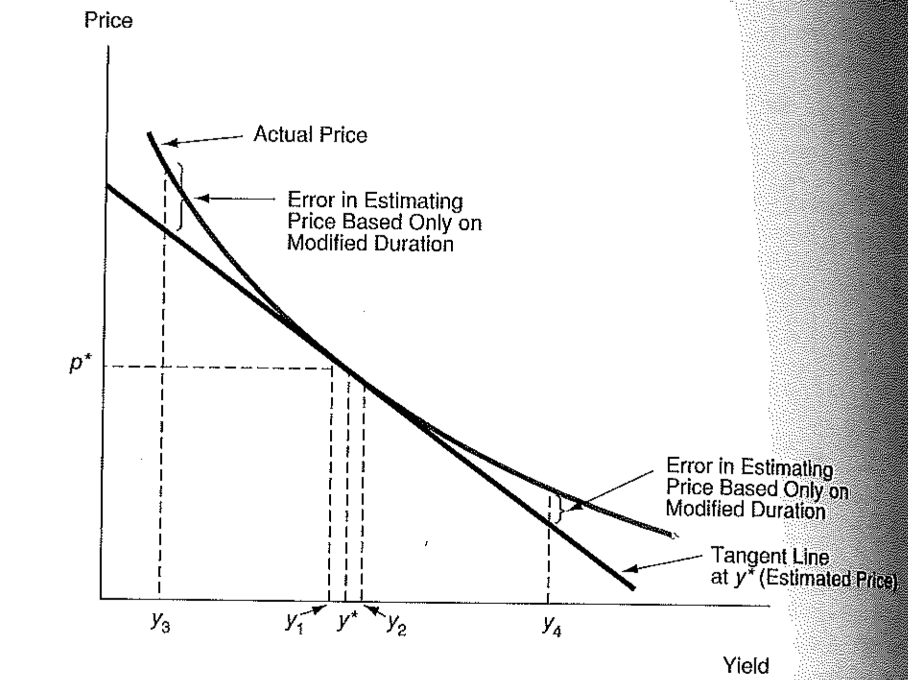


FIGURE 16.9 **PRICE APPROXIMATION USING MODIFIED DURATION**



Source: Frank J. Fabozzi, Mark Pitts, and Ravi E. Dattatreya, "Price Volatility Characteristics of Fixed-Income Securities," in *The Handbook of Fixed-Income Securities*, 4th ed. (Richard D. Irwin, Inc., © 1995) p. 99.

Given this price–yield curve, modified duration is the percentage change in price for a nominal change in yield as follows:[27]

$$D_{mod} = \frac{\frac{dP}{di}}{P}$$

Notice that the dP/di line is tangent to the price–yield curve *at a given yield* as shown in Figure 16.9. For *small* changes in yields (i.e., from y^* to either y_1 or y_2), this tangent straight line gives a good estimate of the actual price changes. In contrast, for larger changes in yields (i.e., from y^* to either y_3 or y_4), the straight line will estimate the new price of the bond at less than the actual price shown by the price–yield curve. This misestimate arises because the modified-duration line is a linear estimate of a curvilinear relationship. Specifically, the estimate using only modified duration will *underestimate* the actual price *increase* caused by a yield decline and *overestimate* the actual price *decline* caused by an increase in yields. This graph, which demonstrates the convexity effect, also shows that price changes are *not* symmetric when yields increase or decrease. As shown, when rates decline, there is a larger price error than when rates increase

[27] In mathematical terms, modified duration is the first differential of this price–yield relationship with respect to yield.

because when yields decline prices rise at an *increasing* rate, while prices decline at a *decreasing* rate when yields rise.

DETERMINANTS OF CONVEXITY Convexity is a measure of the curvature of the price–yield relationship. In turn, because modified duration is the slope of the curve at a given yield, convexity indicates changes in duration. Mathematically, convexity is the second derivative of price with respect to yield (d^2P/di^2) divided by price. Specifically, convexity is the percentage change in dP/di for a given change in yield:

$$\text{Convexity} = \frac{\frac{d^2P}{di^2}}{P}$$

Convexity is a measure of how much a bond's price–yield curve deviates from the linear approximation of that curve. As indicated by Figures 16.7 and 16.9 for *noncallable* bonds, convexity always is a positive number, implying that the price–yield curve lies above the modified duration (tangent) line. Figure 16.8 illustrates the price–yield relationship for two bonds with very different coupons and maturities. (The yields and prices are contained in Table 16.9.)

These graphs demonstrate the following relationship between these factors and the convexity of a bond.

- There is an *inverse* relationship between coupon and convexity (yield and maturity constant).
- There is a *direct* relationship between maturity and convexity (yield and coupon constant).
- There is an *inverse* relationship between yield and convexity (coupon and maturity constant). This means that the price–yield curve is more convex at its lower-yield (upper left) segment.

Therefore, a short-term, high-coupon bond, such as the 12 percent coupon, 3-year bond in Figure 16.8, has very low convexity—it is almost a straight line. In contrast, the zero coupon, 30-year bond has high convexity.

THE MODIFIED DURATION–CONVEXITY EFFECTS In summary, the change in a bond's price resulting from a change in yield can be attributed to two sources: the bond's modified duration and its convexity. The relative effect of these two factors on the price change will depend on the characteristics of the bond (i.e., its convexity) and the size of the yield change. For example, if you are estimating the price change for a 300 basis point change in yield for a zero coupon, 30-year bond, the convexity effect would be fairly large because this bond would have high convexity, and a 300 basis point change in yield is relatively large. In contrast, if you are dealing with only a 10 basis point change in yields, the convexity effect would be minimal because it is a small change in yield. Similarly, the convexity effect would be small for a larger yield change if you are concerned with a bond with small convexity (i.e., a high coupon, short maturity bond) because its price–yield curve is almost a straight line.

In conclusion, modified duration can help you derive an *approximate* percentage bond price change for a given change in interest rates, but you must remember that it only is a good estimate when you are considering small yield changes. You must also consider the convexity effect on price change when you are dealing with large yield changes or when the securities or cash flows have high convexity.

COMPUTATION OF CONVEXITY Again, the formula for computing the convexity of a stream of cash flows looks fairly complex, but it can be broken down into manageable steps. You will recall from our convexity equation above that

$$\text{Convexity} = \frac{\frac{d^2P}{di^2}}{P}$$

In turn,

$$\frac{d^2P}{di^2} = \frac{1}{(1+i)^2}\left[\sum_{t=1}^{n} \frac{CF_t}{(1+i)^t}(t^2 + t)\right]$$

Table 16.13 contains the computations related to this calculation for a 3-year bond with a 12 percent coupon and 9 percent YTM assuming annual flows.

The convexity for this bond is very low because it has a short maturity, high coupon, and high yield. Note that *the convexity of a security will vary along the price–yield curve.* You will get a different convexity at a 3 percent yield than at a 12 percent yield. In terms of the computation, the maturity and coupon will be the same, but you will use a different discount rate that reflects where you are on the curve. This is similar to the earlier observation that *you will get a different modified duration at different points on the price–yield curve* because the slope varies along the curve. You also can see this mathematically because, depending on where you are on the curve, you will be using a different market yield, and the Macaulay and modified durations are inverse to the discount rate.

TABLE 16.13 **COMPUTATION OF CONVEXITY**

$$\text{Convexity} = \frac{d^2P/di^2}{PV \text{ of Cash Flows}} = \frac{d^2P/di^2}{\text{Price}}$$

$$\frac{d^2P}{di^2} = \frac{1}{(1+i)^2}\left[\sum_{t=1}^{n} (t^2 + t)\frac{CF_t}{(1+i)^t}\right]$$

$$\text{Convexity} = \frac{d^2P/di^2}{\text{Price}}$$

Example: 3-Year Bond, 12% Coupon, 9% YTM

(1) Year	(2) CF_t	(3) PV @ 9%	(4) PV CF	(5) $t^2 + t$	(4) × (5)
1	120	.9174	$ 110.09	2	$ 220.18
2	120	.8417	101.00	6	606.00
3	120	.7722	92.66	12	1,111.92
3	1000	.7722	772.20	12	9,266.40
			$1,075.95		$11,204.50

$$\frac{1}{(1+i)^2} = \frac{1}{(1.09)^2} = \frac{1}{1.19} = .84$$

$$\$11{,}204.50 \times .84 = \$9{,}411.78$$

$$\frac{9411.78}{1075.95} = 8.75$$

To compute the price change attributable to the convexity effect after you know the bond's convexity, use this equation:

$$\text{Price Change Due to Convexity} = \tfrac{1}{2} \times \text{Price} \times \text{Convexity} \times (\Delta \text{ in yield})^2$$

Table 16.14 shows the change in bond price considering the duration effect and the convexity effect for an 18-year bond with a 12 percent coupon and 9 percent YTM. For demonstration purposes, we assumed a decline of 100 and 300 basis points (BP) in rates (i.e., 9 percent to 8 percent and 9 percent to 6 percent).

With the 300 BP change, if you considered only the modified-duration effect, you would have *estimated* that the bond went from 126.50 to 158.30 (a 25.14 percent increase), when, in fact, the actual price is closer to 164.41, which is about a *30 percent increase*.

TABLE 16.14 **ANALYSIS OF BOND PRICE CHANGE CONSIDERING DURATION AND CONVEXITY**

Example: 18-Year Bond, 12% Coupon, 9% YTM
Price: 126.50
Modified Duration: 8.38 (D^*)
Convexity: 107.70
Estimate of Price Change Using Duration:
Percent Δ Price = $-D^*$ (Δ in YLD/100)
Estimate of Price Change from Convexity:
Price Change = ½ × Price × Convexity × (Δ in YLD)²

A. Change in Yield: −100 BP

Duration Change: $-8.38 \times \left(\frac{-100}{100}\right) = +8.38\%$

$+8.38\% \times 126.50 = +10.60$

Convexity Change: $\frac{1}{2} \times (126.50) \times 107.70 \times (.01)^2$

$= 63.25 \times 107.70 \times .0001$
$= 6{,}812.03 \times .0001 = .68$

Combined Effect:
126.50
+ 10.60 (Duration)
137.10
+.68 (Convexity)
137.78

B. Change in Yield: −300 BP

Duration Change: $-8.38 \times \left(\frac{-300}{100}\right) = +25.14\%$

$126.50 \times 1.2514 = 158.30\ (+31.80)$

Convexity Effect: $\frac{1}{2} \times (126.50) \times 107.70 \times (.03)^2$

$6.812.03 \times .0009 = 6.11$

Combined Effect:
126.50
+ 31.80 (Duration)
158.30
+6.11 (Convexity)
164.41

NWN/502
Summers/27

DURATION AND CONVEXITY FOR CALLABLE BONDS

The discussion and presentation thus far regarding Macaulay and modified durations and convexity have been concerned with noncallable bonds. A callable bond is different because it provides the issuer with an option to call the bond under certain conditions and pay it off with funds from a new issue sold at a lower yield. Observers will refer to this as a bond with an *embedded option*. We noted earlier that the duration of a bond can be seriously affected by an embedded call option if interest rates decline substantially below a bond's coupon rate. In such a case, the issuer will likely call the bond, which will dramatically change the maturity and the duration of the bond. For example, assume a firm issues a 30-year bond with a 9 percent coupon with a deferred call provision whereby the bond can be called in 6 years at 109 percent of par. If the bond is issued at par, its original *duration to maturity* will be about 11 years. A year later, if rates decline to about 7 percent, its duration to maturity will still be over *10 years* because duration is inversely related to yield and yields have declined. Notably, at a yield of 7 percent, this bond will probably trade at *yield to call* because at a 7 percent yield the firm will likely exercise its option and call the bond in 5 years. Notably, the bond's *duration to first call* would be about *4 years*. Clearly, there is a significant difference between duration to maturity and duration to first call.

To understand the impact of the call feature on the duration and convexity of a bond, it is important to consider what determines the price of a callable bond. A callable bond is a combination of a noncallable bond plus a *call option* that was *sold to the issuer*, which allows the issuer to call the bond under the conditions discussed earlier. Because the call option is owned by the issuer, it has negative value for the investor in the bond. Thus the bondholder's position is:

Long a Callable Bond = Long a Noncallable Bond + A Short Position in a Call Option

Therefore, the value (price) of a callable bond is equal to:

Callable Bond Price = Noncallable Bond Price − Call Option Price

Given this valuation, anything that increases the value of the call option will reduce the value of the callable bond.[28]

OPTION-ADJUSTED DURATION[29] Given these two extreme values of duration to maturity and duration to first call, the investment community derives a duration estimate that is referred to as an option-adjusted or call-adjusted duration based on *the probability that the issuing firm will exercise its call option* for the bond when the bond becomes freely callable. This option-adjusted duration will be somewhere between these two extreme values. Specifically, when interest rates are substantially above the coupon rate, the probability of the bond being called is very small (i.e., the call option has very little value) and the option-adjusted duration will approach the duration to maturity. In contrast, if interest rates decline to levels substantially below the coupon rate, the probability of the bond being called at the first opportunity is very high (i.e., the call option is very valuable and will probably be exercised) and the option-adjusted duration will approach the duration to first call. The bond's option-adjusted duration will be somewhere between these two extremes with the exact option-adjusted duration depending on the level of interest rates relative to the bond's coupon rate.

[28] For a further discussion of the effect of these embedded options, see Frank J. Fabozzi, Mark Pitts, and Ravi E. Dattatreya, "Price Volatility Characteristics of Fixed Income Securities," and Frank J. Fabozzi, Andrew J. Kalotay, and George O. Williams, "Valuation of Bonds with Embedded Options." Both are in Frank J. Fabozzi and T. Dessa Fabozzi, eds., *The Handbook of Fixed-Income Securities*, 4th ed. (Burr Ridge, IL: Irwin Professional Publishing, 1995). Also see Kurt Winkelmann, "Uses and Abuses of Duration and Convexity," *Financial Analysts Journal* 45, no. 5 (September–October 1989): 72–75, and Chapter 14 in Frank J. Fabozzi, *Bond Markets, Analysis and Strategies*, 3rd ed. (Upper Saddle River, NJ: Prentice Hall, 1996).

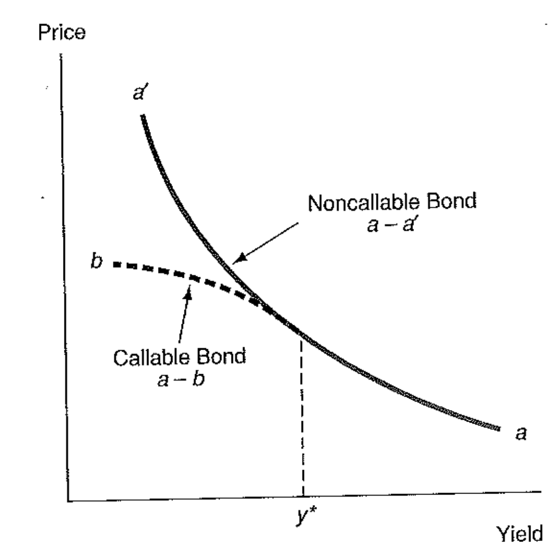
[29] The discussion in this subsection will consider the option-adjusted duration on a conceptual and intuitive basis. For a detailed mathematical treatment, see Dunetz and Mahoney, "Using Duration and Convexity in the Analysis of Callable Bonds."

CONVEXITY OF CALLABLE BONDS Figure 16.10 shows what happens to the price of a callable bond versus the value of a noncallable bond when interest rates increase or decline. Starting from yield y^* (which is close to the par value yield), if interest rates *increase*, the value of the call option declines because at market interest rates that are substantially above the coupon rate, it is unlikely the issuer will want to call the issue. Therefore, the call option has very little value and the price of the callable bond will be similar to the price of a noncallable bond. In contrast, when interest rates *decline* below y^*, there is an increase in the probability that the issuer will want to use the call option—i.e., the value of the call option increases. As a result, the value of the callable bond will deviate from the value of the noncallable bond—i.e., the price of the callable bond will initially not increase as fast as the noncallable bond price and eventually will not increase at all. This is what is shown in curves a–b.

In the case of the noncallable bond, we indicated that it had *positive convexity* because as yields declined, the price of the bond increased at a *faster* rate. With the callable bond, when rates declined, the price increased at a *slower* rate and eventually does not change at all. This pattern of price–yield change for a callable bond is referred to as *negative convexity*.

Needless to say, this price pattern (negative convexity) is one of the risks of a callable bond versus a noncallable bond, especially if there is a chance of declining interest rates.

FIGURE 16.10 **NONCALLABLE AND CALLABLE BOND PRICE–YIELD RELATIONSHIP**



Source: Frank J. Fabozzi, Mark Pitts, and Ravi E. Dattatreya, "Price Volatility Characteristics of Fixed Income Securities," in *The Handbook of Fixed-Income Securities*, 4th ed., edited by Frank J. Fabozzi and T. Dessa Fabozzi (Burr Ridge, IL: Irwin Professional Publishing, 1995). Reprinted by permission of the publisher.

NWN/502
Summers/28

Limitations of Macaulay and Modified Duration

It is important to understand Macaulay and modified duration because of the perspective they provide regarding factors that affect the volatility and interest rate sensitivity of bonds. However, it also is important for bond analysts and portfolio managers to recognize the serious limitations of these measures in the real world. The major limitations are as follows:

First, as noted in the discussion of convexity, the percent change estimates using modified duration only are good for small-yield changes. This was demonstrated in Figure 16.9. As a result, two bonds with equal duration may experience different price changes for large-yield changes—depending on *differences in the convexity* of the bonds.

Second, it is difficult to determine the interest-rate sensitivity of a portfolio of bonds when there is a change in interest rates and the yield curve experiences *a nonparallel shift*. It was noted earlier that the duration of a portfolio is the weighted average of the durations of the bonds in the portfolio. Everything works well as long as all yields change by the same amount—i.e., there is a parallel shift of the yield curve. The problem is, when yields change, the yield curve *seldom* experiences a parallel shift. Assuming a nonparallel shift, which yield do you use to describe the change—the short-, intermediate-, or long-maturity yield? Two portfolios that begin the period with the same duration can have different ending durations and perform very differently, depending on how the yield curve changed (i.e., did it steepen or flatten?) and the composition of the portfolio (i.e., relative to its duration, was it a bullet or a barbell?). Consider the following simple example for two portfolios that have a duration of 4.50 years:

Bond	Coupon	Maturity (Years)	Yield	Modified Duration	Weight
Portfolio A					
A	7.00	4	7.00	2.70	0.555
B	9.00	20	9.00	6.75	0.445
Portfolio B					
C	8.00	10	8.00	4.50	1.000

As shown, the modified durations are equal at the initiation of the portfolio. Assume a nonparallel change in yields where the *yield curve steepens*. Specifically, 4-year yields decline to 6 percent, 10-year yields do not change, and 20-year yields rise to 10 percent. Portfolio B would experience a very small change in value because of stability in yield for 10-year bonds. In contrast, the price for 4-year bonds will experience a small increase (because of small duration) and the value of 20-year bonds will experience a large decline. Overall, the value of portfolio A will decline because of the weight of bond B in the portfolio and its large decline in value due to its large modified duration. Obviously, if the yield curve had flattened or inverted, the barbell portfolio would have benefited from the change. This differential performance because of the change in the shape of the yield curve (i.e., it did not experience a parallel shift) is referred to as *yield curve risk*, which cannot be captured by the traditional duration–convexity presentation.

The third limitation of Macaulay and modified durations involves our initial calculation. We assumed that cash flows from the bond *were not affected by yield changes*—i.e., we assumed option-free bonds. Later, we saw the effect on the computed duration and convexity when we considered the effect of an embedded call option in Figure 16.10. Specifically, we saw that the option-adjusted duration would be some value between the duration to maturity and duration to first call and the specific value would depend on the current

market yield relative to the bond's coupon. Further, we saw that when interest rates declined with an embedded option, the convexity of the bond went from some positive value to *negative* convexity because the price of the callable bond increased at a slower rate or it did not change when the yields declined (i.e., there is *price compression*).

Because of these limitations, practitioners have developed a way to approximate the duration of a bond or any security that will be impacted by a change in interest rates. This is referred to as *effective duration,* which is discussed in the following section.

Effective Duration As noted previously, the purpose of duration is to indicate the price change of an asset to a change in yield—i.e., it is *a measure of the interest-rate sensitivity of an asset.* Because modified duration is based on Macaulay duration, it can provide a reasonable approximation of the interest-rate sensitivity of a bond that experiences a small-yield change and one that is option free—i.e., if yield changes do not change the cash flows for the bond. Unfortunately, the Macaulay and modified duration measures cannot be used for large-yield changes, for assets with embedded options, or for other assets that are affected by variables other than interest rates such as common stocks or real estate.

To overcome these limitations, practitioners use *effective duration*, a direct measure of the interest-rate sensitivity of a bond or any asset where it is possible to observe the market prices surrounding a change in interest rates. As we will demonstrate, using this measure we can derive negative durations (which is not mathematically possible with Macaulay), or durations that are longer than the maturity of the asset (not possible with Macaulay). The concept is best described by recalling the formula to determine the percentage price change for a bond using modified duration as follows:

$$\%\Delta\text{Price} = -D^* \times (\Delta R)$$

where

D^* = the modified Macaulay duration
ΔR = the change in interest rates in basis points divided by 100.

The typical assumption is that we know D^* and ΔR and can solve for the approximate percent price change. Given this relationship, we can solve for D^* as follows:

$$-D^* = \frac{\%\Delta\text{Price}}{\Delta R}$$

When we solve for it this way, it is no longer D^* (modified duration), but D_E—effective duration. Given this formulation, if you observe a change in interest rates (ΔR) and the change in the price of an asset during the same time period, you can solve for the effective duration of the asset. Consider the following simple example.

- Interest rates decline by 200 b.p.
- The price of a bond increases by 10 percent

$$D_E - \frac{10}{-200/100} = -\frac{10}{-2} = 5$$

NWN/502
Summers/29

Therefore, the change in price coincident with a change in interest rates indicates that the bond has an effective duration (D_E) of 5. This is a direct measure of the bond's interest-rate sensitivity. Notably, thinking of duration in this way, it is not appropriate to describe it as a measure of time (i.e., in years). As noted, it is a measure of interest rate sensitivity and you should think of it as *the approximate percentage change in price for a 100-basis-point change in interest rates.*

EFFECTIVE DURATION GREATER THAN MATURITY Because effective duration is simply interest-rate sensitivity, it is possible to have an asset that is highly levered such that its interest-rate sensitivity exceeds its maturity. For example, there are 5-year, collateralized mortgage obligations (CMOs) that are highly levered and their prices will change by 15 percent to 20 percent when interest rates change by 100 basis points. Using the formula discussed, this would imply an effective duration of 15 or 20 for a 5-year maturity security.

NEGATIVE EFFECTIVE DURATION We know from the formula for Macaulay duration that it is not possible to compute a negative duration. Further, in the calculation for price volatility where we use $-D^*$ to reflect the negative relationship between price changes and interest rate changes for *option-free bonds*. At the same time, we know that when we leave the world of option-free bonds and consider bonds with embedded options, it is possible to envision cases where bond prices move in the same direction as yields, which implies negative duration. A prime example would be mortgage-backed securities where a significant decline in interest rates will cause a substantial increase in refinancing prepayments by homeowners, which will reduce the value of these bonds to holders. Therefore, you would see a decline in interest rates *and* a decline in the price of these mortgage-backed bonds, which implies *negative duration.*

EFFECTIVE DURATION FOR COMMON STOCK If one considers the Macaulay duration of common stock, it is possible to envision a fairly high number because you are dealing with a perpetuity, and some growth stocks pay low dividends for many years. The values derived by Reilly and Sidhu, using various assumptions of price and growth, ranged from 10 years to 20 years.[30] In contrast, using effective duration one gets very different results.

Because we are dealing with the interest-rate sensitivity of an asset, it is possible to compute an effective duration for common stock that is much lower than implied by Macaulay duration and it is more variable. Observing a change in interest rates and the accompanying change in stock prices would indicate the interest-rate sensitivity of stocks. Leibowitz conducted such an analysis and derived a rolling, one-year effective duration for the S&P 500 that ranged *from about zero to almost 7.*[31] Because we are measuring interest-rate sensitivity over time, you would expect changes in the interest-rate sensitivity of common stocks over time because the correlation between stocks and bonds varies. In addition you might anticipate significant differences in the effective duration for alternative stocks. For example, you would expect a large difference in the interest-rate sensitivity (effective duration) of a banking or utility stock (which is very interest rate sensitive) compared to the effective duration of a technology growth stock where its value is based more on changes in its growth expectations than interest rates.

[30] Frank K. Reilly and Rupinder Sidhu, "The Many Uses of Bond Duration," *Financial Analysts Journal* 36, no. 4 (July–August 1980): 58–72.

[31] Martin L. Leibowitz, *New Perspective on Asset Allocation* (Charlottesville, VA: The Research Foundation of the Institute of Chartered Financial Analysts, 1987).

Summary

- The value of a bond equals the present value of all future cash flows accruing to the investor. Cash flows for the conservative bond investor include periodic interest payments and principal return; cash flows for the aggressive investor include periodic interest payments and the capital gain or loss when the bond is sold prior to its maturity. Bond investors can maximize their yields by accurately estimating the level of interest rates, and more importantly, by estimating changes in interest rates and yield spreads. Similarly, they must compare coupon rates, maturities, and call features of alternative bonds.
- There are five bond yield measures: nominal yield, current yield, promised yield to maturity, promised yield to call, and realized (horizon) yield. The promised YTM and promised YTC equations include the interest-on-interest (or coupon reinvestment) assumption. For the realized (horizon) yield computation, the investor estimates the reinvestment rate and may need to estimate the future selling price for the bond. The fundamental determinants of interest rates are a real risk-free rate, the expected rate of inflation, and a risk premium.
- The yield curve (or the term structure of interest rates) shows the relationship between the yields on a set of comparable bonds and the term to maturity. Based upon this yield curve it is possible to derive a theoretical spot rate curve. In turn, these spot rates can be used to value bonds using an individual spot rate for each cash flow. In addition, these spot rates imply investor expectations about future rates referred to as forward rates. Yield curves exhibit four basic patterns. Three theories attempt to explain the shape of the yield curve: the expectations hypothesis, the liquidity preference hypothesis, and the segmented market hypothesis.
- It is important to understand what causes changes in interest rates and how these changes in rates affect the prices of bonds. Differences in bond price volatility are mainly a function of differences in yield, coupon, and term to maturity. There are three duration measures that have been used as measures of bond price volatility or interest-rate sensitivity. The Macaulay duration measure incorporates coupon, maturity, and yield in one measure and an adaptation of it (modified duration) provides an estimate of the response of bond prices to changes in interest rates under certain assumptions. Because modified duration provides a straight-line estimate of the curvilinear price–yield function, you must consider modified duration together with the convexity of a bond for large changes in yields and/or when dealing with securities that have high convexity. It is shown that the call feature on a bond can have a significant impact on its modified duration (the call feature can shorten it dramatically) and on its convexity (the call feature can change the convexity from a positive value to a negative value). Following a discussion of some of the limitations of Macaulay and modified durations as measures of interest-rate sensitivity, we present the concept of effective duration, which is a direct measure of interest-rate sensitivity—i.e., it is the approximate percentage change in price for a 100-basis-point change in interest rates. Notably, effective duration allows for durations longer than maturity, negative duration, and duration estimates for common stock.

Given the background in bond valuation and the factors that influence bond value and bond return volatility, we are ready to consider how to build a bond portfolio that is consistent with our goals and objectives. Bond portfolio analysis is the topic for Chapter 17.

Questions

1. Why does the present value equation appear to be more useful for the bond investor than for the common stock investor?
2. What are the important assumptions made when you calculate the promised yield to maturity? What are the assumptions when calculating promised YTC?
3. a. Define the variables included in the following model:

$$i = (\text{RFR}, I, \text{RP})$$

   b. Assume that the firm whose bonds you are considering is not expected to break even this year. Discuss which factor will be affected by this information.
4. We discussed three alternative hypotheses to explain the term structure of interest rates. Briefly discuss the three hypotheses and indicate which one you think best explains the alternative shapes of a yield curve.

CFA

5. *CFA Examination I (June 1982)*
   a. Explain what is meant by *structure of interest rates*. Explain the theoretical basis of an upward-sloping yield curve. [8 minutes]
   b. Explain the economic circumstances under which you would expect to see the inverted yield curve prevail. [7 minutes]
   c. Define "real" rate of interest. [2 minutes]
   d. Discuss the characteristics of the market for U.S. Treasury securities. Compare it to the market for AAA corporate bonds. Discuss the opportunities that may exist in bond markets that are less than efficient. [8 minutes]
   e. Over the past several years, fairly wide yield spreads between AAA corporates and Treasuries have occasionally prevailed. Discuss the possible reasons for this. [5 minutes]

CFA

6. *CFA Examination III (June 1982)*
   As the portfolio manager for a large pension fund, you are offered the following bonds:

	Coupon	Maturity	Price	Call Price	Yield to Maturity
Edgar Corp. (new issue)	14.00%	2002	$101.3/4	$114	13.75%
Edgar Corp. (new issue)	6.00	2002	48.1/8	103	13.60
Edgar Corp. (1972 issue)	6.00	2002	48.7/8	103	13.40

Assuming that you expect a decline in interest rates over the next 3 years, identify and justify which of these bonds you would select. [10 minutes]

7. You expect interest rates to decline over the next six months.
   a. Given your interest rate outlook, state what kinds of bonds you want in your portfolio in terms of duration and explain your reasoning for this choice.
   b. You must make a choice between the following three sets of noncallable bonds. In each case, select the bond that would be best for your portfolio given your interest rate outlook and the consequent strategy set forth in Part a. In each case briefly discuss why you selected the bond.

	Maturity	Coupon	Yield to Maturity
Case 1: Bond A	15 years	10%	10%
Bond B	15 years	6%	8%
Case 2: Bond C	15 years	6%	10%
Bond D	10 years	8%	10%
Case 3: Bond E	12 years	12%	12%
Bond F	15 years	12%	8%

8. At the present time, you expect a decline in interest rates and must choose between two portfolios of bonds with the following characteristics:

	Portfolio A	Portfolio B
Average maturity	10.5 years	10.0 years
Average YTM	7%	10%
Modified duration	5.7 years	4.9 years
Modified convexity	125.18	40.30
Call features	Noncallable	Deferred call features that range from 1 to 3 years

Select one of the portfolios and discuss three factors that would justify your selection.

9. The Chartered Finance Corporation has issued a bond with the following characteristics:

Maturity—25 years
Coupon—9%
Yield to maturity—9%
Callable—after 3 years @ 109
Duration to maturity—8.2 years
Duration to first call—2.1 years

   a. Discuss the concept of call-adjusted duration and indicate the approximate value (range) for it at the present time.
   b. Assuming interest rates increase substantially (i.e., to 13 percent), discuss what will happen to the call-adjusted duration and the reason for the change.
   c. Assuming interest rates decline substantially (i.e., they decline to 4 percent), discuss what will happen to the bond's call-adjusted duration and the reason for the change.
   d. Discuss the concept of negative convexity as it relates to this bond.

CFA

10. *CFA Examination I (1990)*
    Duration may be calculated by *two* widely used methods. Identify these *two* methods, and briefly discuss the primary differences between them. [5 minutes]

CFA

11. *CFA Examination II (1995)*
    Option-adjusted duration and *effective duration* are alternative measures used by analysts to evaluate fixed-income securities with embedded options.
    Briefly describe *each* measure and how to apply *each* to the evaluation of fixed-income securities with embedded options. [8 minutes]

CFA

12. *CFA Examination II (1995)*
    As a portfolio manager, during a discussion with a client, you explain that historical return and risk premia of the type presented in the following Table are frequently used in forming estimates of future returns for various types of financial assets. While such historical data are helpful in forecasting returns, most users know that history is an imperfect guide to the future. Thus, they recognize that there are reasons why these data should be adjusted if they are to be employed in the forecasting process.

U.S. HISTORICAL RETURN AND RISK PREMIA (1926–94)

	Per Year
Inflation rate	3.0%
Real interest rate on Treasury bills	0.5%
Maturity premium of long Treasury bonds over Treasury bills	0.8%
Default premium of long corporate bonds over long Treasury bonds	0.6%
Risk premium on stock over long Treasury bonds	5.6%
Return on Treasury bills	3.5%
Return on long corporate bonds	4.9%
Return on large-capitalization stocks	9.9%

   a. As shown in the Table the historical real interest rate for Treasury bills was 0.5% per year and the maturity premium on Treasury bonds over Treasury bills was 0.8%. Briefly describe and justify *one* adjustment to *each* of these two data items that should be made before they can be used to form expectations about future real interest rates and Treasury bond maturity premia. [6 minutes]
   b. You recognize that even adjusted historical economic and capital markets data may be of limited use when estimating future returns. Independent of your Part A response, briefly describe *three* key circumstances that should be considered when forming expectations about future returns. [8 minutes]

13. *CFA Examination I (1992)*
A portfolio manager at Superior Trust Company is structuring a fixed-income portfolio to meet the objectives of a client. This client plans on retiring in 15 years and wants a substantial lump sum at that time. The client has specified the use of AAA-rated securities.
The portfolio manager compares coupon U.S. Treasuries with zero coupon stripped U.S. Treasuries and observes a significant yield advantage for the stripped bonds.

Maturity	Coupon U.S. Treasuries	Zero Coupon Stripped U.S. Treasuries
3 year	5.50%	5.80%
5 year	6.00%	6.60%
7 year	6.75%	7.25%
10 year	7.25%	7.60%
15 year	7.40%	8.80%
30 year	7.75%	7.75%

Briefly discuss *two* reasons why zero coupon stripped U.S. Treasuries could yield more than coupon U.S. Treasuries with the same final maturity. [5 minutes]

14. *CFA Examination II (1993)*
a. In terms of option theory, explain the impact on the offering yield of adding a call feature to a proposed bond issue. [5 minutes]
b. Explain the impact on *both* bond duration and convexity of adding a call feature to a proposed bond issue. [10 minutes]
Assume that a portfolio of corporate bonds is managed to maintain targets for modified duration and convexity.
c. Explain how the portfolio could include *both* callable and non-callable bonds while maintaining the targets. [5 minutes]
d. Describe *one* advantage and *one* disadvantage of including callable bonds in this portfolio. [5 minutes]

Problems

1. Four years ago, your firm issued $1,000 par, 25-year bonds, with a 7 percent coupon rate and a 10 percent call premium.
a. If these bonds are now called, what is the *approximate* yield to call for the investors who originally purchased them?
b. If these bonds are now called, what is the *actual* yield to call for the investors who originally purchased them at par?
c. If the current interest rate is 5 percent and the bonds were not callable, at what price would each bond sell?
2. Assume that you purchased an 8 percent, 20-year, $1,000 par, semiannual payment bond priced at $1,012.50 when it has 12 years remaining until maturity. Compute:
a. Its approximate yield to maturity
b. Its actual yield to maturity
c. Its yield to call if the bond is callable in 3 years with an 8 percent premium
3. Calculate the duration of an 8 percent, $1,000 par bond that matures in 3 years if the bond's YTM is 10 percent and interest is paid semiannually.
a. Calculate this bond's modified duration.
b. Assuming the bond's YTM goes from 10 percent to 9.5 percent, calculate an estimate of the price change.
4. Two years ago, you acquired a 10-year zero coupon, $1,000 par value bond at a 12 percent YTM. Recently you sold this bond at an 8 percent YTM. Using semiannual compounding, compute the annualized horizon return for this investment.

5. A bond for the Webster Corporation has the following characteristics:

Maturity—12 years
Coupon—10%
Yield to maturity—9.50%
Macaulay duration—5.7 years
Convexity—48
Noncallable

a. Calculate the approximate price change for this bond using only its duration assuming its yield to maturity increased by 150 basis points. Discuss the impact of the calculation, including the convexity effect.
b. Calculate the approximate price change for this bond (using only its duration) if its yield to maturity declined by 300 basis points. Discuss (without calculations) what would happen to your estimate of the price change if this was a callable bond.
6. *CFA Examination I (1992)*
The table below shows selected data on a German government bond (payable in Deutschemarks) and a U.S. government bond. Identify the components of return and calculate the total return in U.S. dollars for both of these bonds for the year 1991. Show the calculations for *each* component. (Ignore interest on interest in view of the short time period.) [8 minutes]

		MARKET YIELD		Modified	EXCHANGE RATE (DM/$U.S.)	
	Coupon	1/1/91	1/1/92	Duration	1/1/91	1/1/92
German Government Bond	8.50%	8.50%	8.00%	7.0	1.55	1.50
U.S. Government Bond	8.00%	8.00%	6.75%	6.5	—	—

7. *CFA Examination I (1993)*
Philip Morris has issued bonds that pay semi-annually with the following characteristics:

Coupon	Yield-to-Maturity	Maturity	Macaulay Duration
8%	8%	15 years	10 years

a. Calculate modified duration using the information above. [5 minutes]
b. Explain why modified duration is a better measure than maturity when calculating the bond's sensitivity to changes in interest rates. [5 minutes]
c. Identify the direction of change in modified duration if:
(i) the coupon of the bond were 4%, not 8%
(ii) the maturity of the bond were 7 years, not 15 years [5 minutes]
d. Define convexity and explain how modified duration *and* convexity are used to approximate the bond's percentage change in price, given a change in interest rates. [5 minutes]
8. *CFA Examination I (1993)*
You are a U.S. investor considering purchase of one of the following securities. Assume that the currency risk of the German government bond will be hedged, and the six-month discount on Deutschemark forward contracts is –0.75% versus the U.S. dollar.

Bond	Maturity	Coupon	Price
U.S. government	June 1, 2003	6.50%	100.00
German government	June 1, 2003	7.50%	100.00

Calculate the expected price change required in the German government bond which would result in the two bonds having equal total returns in U.S. dollars over a six-month horizon. [8 minutes]

9. *CFA Examination II (1990)*
The following are the average yields on U.S. Treasury bonds at two different points in time:

	Yield-to-Maturity	
Term to Maturity	January 15, 19XX	May 15, 19XX
1 year	7.25%	8.05%
2 years	7.50%	7.90%
5 years	7.90%	7.70%
10 years	8.30%	7.45%
15 years	8.45%	7.30%
20 years	8.55%	7.20%
25 years	8.60%	7.10%

a. Assuming a pure expectations hypothesis, define a forward rate. Describe how you would calculate the forward rate for a three-year U.S. Treasury bond two years from May 15, 19XX using the actual term structure above. [3 minutes]
b. Discuss how *each* of the *three* major term structure hypotheses could explain the January 15, 19XX term structures shown above. [6 minutes]
c. Discuss what happened to the term structure over the time period and the effect of this change on U.S. Treasury bonds of 2 years *and* 10 years. [5 minutes]
d. Assume that you invest solely on the basis of yield spreads, and in January 19XX acted upon the expectation that the yield spread between 1-year and 25-year U.S. Treasuries would return to a more typical spread of 170 basis points. Explain what you would have done on January 15, 19XX, and describe the result of this action based upon what happened between January 15, 19XX and May 15, 19XX. [7 minutes]

10. *CFA Examination II (1992)*
a. Using the information in the table below calculate the projected price change for Bond B if the yield-to-maturity for this bond falls by 75 basis points. [7 minutes]
b. Describe the shortcoming of analyzing Bond A strictly to call or to maturity. Explain an approach to remedy this shortcoming. [6 minutes]

MONTICELLO CORPORATION BOND INFORMATION

	Bond A (Callable)	Bond B (Non-Callable)
Maturity	2002	2002
Coupon	11.50%	7.25%
Current price	125.75	100.00
Yield-to-maturity	7.70%	7.25%
Modified duration to maturity	6.20	6.80
Convexity to maturity	.50	.60
Call date	1996	—
Call price	105	—
Yield to call	5.10%	—
Modified duration to call	3.10	—
Convexity to call	.10	—

11. *CFA Examination II (1992)*
U.S. Treasuries represent a significant holding in Monticello's pension portfolio. You decide to analyze the yield curve for U.S. Treasury Notes.

a. Using the data in the table below, calculate the five-year spot and forward rates assuming annual compounding. Show calculations. [8 minutes]

U.S. TREASURY NOTE YIELD CURVE DATA

Years to Maturity	Par Coupon Yield-to-Maturity	Calculated Spot Rates	Calculated Forward Rates
1	5.00	5.00	5.00
2	5.20	5.21	5.42
3	6.00	6.05	7.75
4	7.00	7.16	10.56
5	7.00	☐	☐

b. Define and describe *each* of the following *three* concepts:
- Yield-to-maturity
- Spot rate
- Forward rate.

Explain how these *three* concepts are related. [9 minutes]

You are considering the purchase of a zero-coupon U.S. Treasury Note with four years to maturity.

c. Based on the above yield curve analysis, calculate *both* the expected yield-to-maturity and the price for the security. Show calculations. [8 minutes]

12. *CFA Examination III (1992)*
Emily Maguire, manager of the actively managed non-government bond portion of PTC's pension portfolio, has received a fact sheet containing data on a new security offering. It will be a bond issued by a U.S. corporation but denominated in Australian dollars (A$), with both principal and interest payable in that currency.

The terms of the offering made in June, 1992 are as follows:
- Issuer—Student Loan Marketing Association (SLMA—a U.S. Government Sponsored Corporation)
- Rating—AAA
- Coupon Rate—8.5% payable quarterly
- Price—Par
- Maturity—June 30, 1997 (non-callable)
- Principal and interest payable in Australian dollars (A$)

As an alternative, Maguire finds that five-year U.S.$-pay notes issued by SLMA yield 6.75%.

She prepares an analysis directed at several specific questions, beginning with the following table of economic data for Australia and the United States.

	United States			Australia		
Major Economic Indicators	1990	1991	1992(E)	1990	1991	1992(E)
Real GNP (annual change)	1.1%	−0.5%	2.2%	1.6%	−0.5%	3.0%
Consumer expenditures (annual change)	0.9%	0.0%	1.0%	1.1%	−0.2%	2.0%
Inflation (annual change)	5.4%	4.2%	3.4%	7.3%	3.2%	3.9%
Long-bond yield (end-of-year)	8.1%	7.2%	7.0%	9.8%	10.0%	10.2%
Trade balance (U.S. $ billions)	−100	−83	−80	−30	−20	−25

Assuming that interest rates fall 100 basis points in both the U.S. and Australian markets over the next year, identify which of these two bonds will increase the most in value, and justify your answer. [7 minutes]

13. *CFA Examination II (1993)*
The following table shows yields to maturity on U.S. Treasury securities as of January 1, 1993.

Term to Maturity	Yield to Maturity
1 year	3.50%
2 years	4.50%
3 years	5.00%
4 years	5.50%
5 years	6.00%
10 years	6.60%

a. Based on the data in the table, calculate the implied forward one-year rate of interest at January 1, 1996. [5 minutes]
b. Describe the conditions under which the calculated forward rate would be an unbiased estimate of the one-year spot rate of interest at January 1, 1996. [5 minutes]

Assume that one year earlier, at January 1, 1992, the prevailing term structure for U.S. Treasury securities was such that the implied forward one-year rate of interest at January 1, 1996, was significantly higher than the corresponding rate implied by the term structure at January 1, 1993.

c. On the basis of the pure expectations theory of the term structure, briefly discuss two factors that could account for such a decline in the implied forward rate. [8 minutes]

Multiple scenario forecasting frequently makes use of information from the term structure of interest rates.

d. Briefly describe how the information conveyed by this observed decrease in the implied forward rate for 1996 could be used in making a multiple scenario forecast. [5 minutes]

14. *CFA Examination III (1993)*
TMP is working with the officer responsible for the defined-benefit pension plan of a U.S. company. She has come to the firm for advice on what she calls "the key elements of non-U.S. dollar fixed-income investing."

The following information, based on TMP's assessment of the Italian market, has been developed to illustrate the process by which market and currency expectations are integrated.

ITALIAN GOVERNMENT SECURITIES DATA

Security	Modified Duration	Current Price	Current Yield to Maturity	Expected Yield to Maturity in 3 Months
Bill	0.25	100.00	12.50%	12.50%
Note	6.00	100.00	10.00%	9.00%

LIRA/$(US) EXCHANGE RATE

Current Rate	Expected Rate in 3 Months
L1500/$1.00 (US)	L1526/$1.00 (US)

Based on the information provided above, calculate the expected return (in U.S. dollars) on each security over the three-month period. [9 minutes]

15. *CFA Examination I (1994)*
Bonds of Zello Corporation with a par value of $1,000 sell for $960, mature in five years, and have a 7% annual coupon rate paid semiannually.
a. Calculate the:
   (i) current yield;
   (ii) yield-to-maturity (to the nearest whole percent, i.e., 3%, 4%, 5%, etc.); *and*
   (iii) horizon yield (also called total return) for an investor with a three year holding period and a reinvestment rate of 6% over the period. At the end of three years the 7% coupon bonds with two years remaining will sell to yield 7%.

   Show your work. [9 minutes]
b. Cite *one* major shortcoming for *each* of the following fixed-income yield measures:
   (i) current yield;
   (ii) yield to maturity; *and*
   (iii) horizon yield (also called total return). [6 minutes]

16. *CFA Examination I (1994)*
During 1990, Disney issued $2.3 billion face value of zero-coupon subordinated notes which resulted in gross proceeds of $965 million. The notes:
- mature in 2005;
- can be exchanged for cash by the note holder at any time for the U.S. dollar equivalent of the current market value of 19.651 common shares of Euro Disney per $1,000 face value of notes; and
- are callable at any time at their issuance price plus accrued interest.

On March 11, 1993 Disney called the notes at a price of $483.50 which is equivalent to a yield to maturity of 6%. On the call date, Euro Disney common stock traded at a price of 86.80 French francs per share and the currency exchange rate for U.S. dollars ($US) to French francs (Ffr) was:

	$US/Ffr	Ffr/$US
Exchange rate:	.1761	5.6786

a. Calculate, as of the call date:
   (i) the price of a share of Euro Disney expressed in U.S. dollars; *and*
   (ii) the exchange value (conversion value) of a $1,000 face value note in U.S. dollars. [6 minutes]
b. On July 21, 1993, Disney issued, at par, $300 million of 100-year bonds with a coupon rate of 7.55%. The bonds are callable in 30 years at 103.02. From Disney's point of view, state *three* disadvantages of calling the zero-coupon notes and effectively replacing part of that debt capital with the issue of 100-year bonds. [8 minutes]

17. *CFA Examination II (1994)*
Table 1 below shows the characteristics of two annual pay bonds from the same issuer with the same priority in the event of default, and Table 2 below displays spot interest rates. Neither bond's price is consistent with the spot rates.
Using the information in Tables 1 and 2, recommend *either* Bond A *or* Bond B for purchase. Justify your choice. [10 minutes]

TABLE 1 BOND CHARACTERISTICS

	Bond A	Bond B
Coupons	Annual	Annual
Maturity	3 years	3 years
Coupon Rate	10%	6%
Yield-to-maturity	10.65%	10.75%
Price	98.40	88.34

TABLE 2 SPOT INTEREST RATES

Term	Spot Rates (Zero Coupon)
1 year	5%
2 year	8%
3 year	11%

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Chapter 17

Bond Portfolio Management Strategies

This chapter answers the following questions:

- What are the four major alternative bond portfolio management strategies available?
- What are the two specific strategies available within the passive portfolio management category?
- What are the five alternative strategies available within the active bond portfolio management category?
- What is meant by matched-funding techniques and what are the four specific strategies available in this category?
- What are the major contingent procedure strategies that are also referred to as structured active management strategies?
- What are the implications of capital market theory for those involved in bond portfolio management?
- What is the evidence on the efficient market hypothesis as it relates to bond markets?
- What are the implications of efficient market studies for those involved in bond portfolio management?

In this chapter, we shift attention from bond valuation and analysis to the equally important bond portfolio management strategies. In the first section, we discuss the alternative portfolio management strategies. This includes a detailed consideration of the four major strategies: passive management, active management, matched funding techniques, and structured active management. Next, we consider the implications of capital market theory and bond market efficiency on bond portfolio management.

ALTERNATIVE BOND PORTFOLIO STRATEGIES

Bond portfolio management strategies can be divided into four groups:[1]

1. Passive portfolio strategies
   a. Buy and hold
   b. Indexing
2. Active management strategies
   a. Interest rate anticipation
   b. Valuation analysis
   c. Credit analysis
   d. Yield spread analysis
   e. Bond swaps

[1] This breakdown benefitted from the discussion in Martin L. Leibowitz, "The Dedicated Bond Portfolio in Pension Funds—Part I: Motivations and Basics," *Financial Analysts Journal* 42, no. 1 (January–February 1986): 61–75.

BEFORE THE

PUBLIC UTILITY COMMISSION OF OREGON

NW Natural

Exhibit 503 of Barbara Summers

UM 1744
Carbon Emission Reduction Program
Combined Heat & Power (CHP)

Pew Center on Global Climate Change and ICF International's Survey of Corporate Energy Efficiency Strategies

October 16, 2015

A Survey of Corporate Energy Efficiency Strategies

William Prindle, ICF International
Andre de Fontaine, Pew Center on Global Climate Change

ABSTRACT

This paper summarizes the results of a 2009 survey of corporate energy efficiency strategies, conducted by the Pew Center on Global Climate Change. Forty-eight companies, ranging in size from $8 billion to $99 billion in revenues, completed the survey. Key results included an average energy savings target of 20%, or 2.2% on an annualized basis. The three leading motivations for companies' energy efficiency strategies were reducing carbon footprint, responding to rising energy prices, and demonstrating commitment to corporate social responsibility. 60% of respondents had full-time energy managers, 87% built energy performance into the compensation review systems for facility/plant management, and 38% reported energy performance criteria at the senior management level. Almost all respondents used specific financial criteria for energy efficiency investments, simple payback and internal rate of return (IRR) being the most common. Simple payback criteria were mostly three years or less, though two were as high as 5 years. IRR criteria were mostly in the 10-15% range, though one reported a 35% IRR threshold. Respondents also reported a variety of qualitative factors affecting their internal operations, supply chains, and product and services, and summarized the lessons learned and ongoing needs for their energy efficiency strategies.

Background

The survey's principal objective was to gather quantitative data, and identify management practices as well as trends in corporate energy efficiency strategies. It is a key element of a broader Pew Center study on best practices in corporate energy efficiency strategies, whose goal is to highlight the most effective methods used by companies today to reduce their energy consumption and lower their related greenhouse gas emissions. It encompasses management approaches to improving energy efficiency, including issues such as organizational structures, financial mechanisms, and employee compensation systems that corporations put in place to drive superior energy performance. The survey results will be combined with a set of case studies in a larger report to be published in late 2009 or early 2010. The report, and related communications activities, is being funded by a three-year, $1.4 million grant from Toyota.

With concerns growing over climate change and future energy price increases, most, if not all, companies stand to benefit from a renewed focus on energy efficiency. By cataloging and describing best practices in corporate energy efficiency, the Pew Center report is intended to serve as a resource to other companies seeking to develop new, or improve upon existing, energy efficiency programs. The report builds upon existing Pew Center research that provides practical guidance to companies seeking to manage the risks and maximize the opportunities associated with the global transition to a low-carbon economy. Past Pew Center reports and white papers have examined corporate climate change strategies, the development of corporate greenhouse gas emissions inventories and reduction targets, adaptation planning for businesses, and the use of carbon offsets.[1]

Sample Design and Response Rate

To get at best practices among industry leaders, the survey sample was drawn from major companies with a demonstrated commitment to climate and energy issues. We deliberately sought larger companies with strong energy/climate commitments, because the goal is to elicit best practices, not average practices. In this sense, the sample is intentionally not representative of the U.S. corporate population. With that objective, we drew the sample mainly from members of business-NGO and/or government-NGO partnership programs on climate change/sustainability. Included in the sample were all 43 of the companies in the Pew Center's Business Environmental Leadership Council (BELC), the largest U.S.-based association of companies dedicated to business and policy solutions to climate change. An additional 51 companies were pulled from such organizations as the U.S. Climate Action Partnership, Climate Group, World Wildlife Funds's Climate Savers, U.S. Environmental Protection Agency's Climate Leaders, and the World Business Council on Sustainable Development. Most of these companies are U.S.-based, though many operate globally; the survey covers respondents' full global operations.

ICF International's Survey Research Center programmed the questionnaire into an on-line instrument, and the Pew Center distributed it via e-mail to the 95 companies in January 2009. Prospective participants received a link to the on-line survey instrument, unique user names and passwords, and a pdf copy of the questionnaire. In all, a total of 48 companies completed the survey, a response rate of approximately 53 percent.

Survey Instrument

The instrument contained a little over 60 questions split into the following sections: general company information; overall strategy; risk management and finance; specific initiatives (internal operations, supply chain considerations, and products and services); and lessons learned. Key questions centered on organizational issues, such as internal champions in establishing efficiency programs; financial issues, such as the financing of efficiency projects and their role in competing with other priorities; and broader "lessons learned," such as major challenges in developing efficiency programs, and the methods by which those challenges were overcome.

Respondent Characteristics

Respondents ranged from semiconductor manufacturers to electric utilities, medical suppliers, chemical manufacturers, beverage companies, apparel makers, airlines, insurance companies, and heavy machinery manufacturers. This sample thus represents a representative range of companies across many different sectors of the economy. Key statistics included:

[1] All Pew Center reports are available for download at www.pewclimate.org.

- Revenues—Ranged from under $8 billion to $99 billion, with an average of just under $29 billion
- Energy costs—Based on the 21 respondents who reported this data, total company energy costs ranged from $25 million to $27 billion, with an average of just under $ 2 billion

Views on Climate Policy and Energy Prices

Almost all participants (98%) believe that comprehensive legislation mandating reductions in greenhouse gas emissions will be enacted in the U.S. More than half of those (57%) believe legislation will be enacted within two years, the remainder within four years.

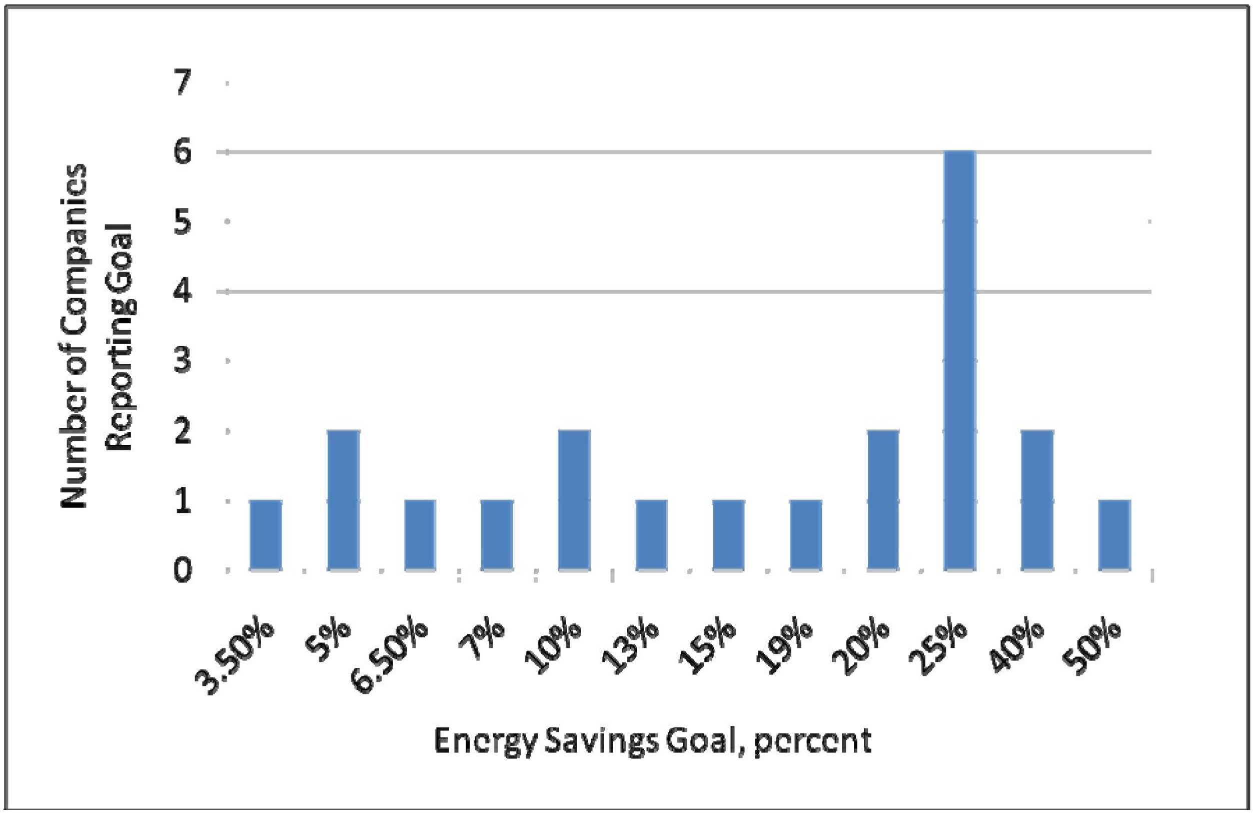
Respondents were also asked where they expect energy prices to be by 2014, using world oil prices as a general proxy. About 5% think prices will stay below $75/barrel for the next five years; 44% believe prices will rise to the $75-99 range, and over half believe oil will exceed $100/barrel by 2014. The U.S. Department of Energy's 2009 Annual Energy Outlook projects a 2014 price of about $104/barrel for crude oil imported by U.S. refiners.[2] Respondents' estimates thus come fairly close to the U.S. official forecast. It is also relevant to point out that prior to the 2006 Annual Energy Outlook, oil price forecasts for 2014 did not exceed $27/barrel. Price expectations have thus risen rapidly in just four years.

Energy Efficiency Goals

One of the survey's main objectives was to obtain companies' quantitative goals for reducing energy usage or costs, using specific metrics. Twenty-one companies in the sample supplied quantitative goal information. The mean energy savings goal was 20%; however, the responses ranged from 3.5% to 50%. It is also important to understand the context for these percentages, in terms of timeframe and metrics; we therefore asked companies to supply the target year for the savings goal, the base year against which it was measured, and the metric in which the goal was expressed. The mean base year was 2003, and the mean target year was 2013. For those who reported a percentage savings target as well as a base year and a target year, the annualized savings percentage was 2.2%; in other words, the average company's target called for just over 2% energy savings per year, over about a 10-year period. A chart showing the range of reported savings targets is shown in Figure 1.

[2] U.S. Department of Energy. Energy Information Administration. *2009 Annual Energy Outlook.* http://www.eia.doe.gov/oiaf/aeo/aeoref_tab.html

Figure 1. Range of Reported Energy Savings Goals

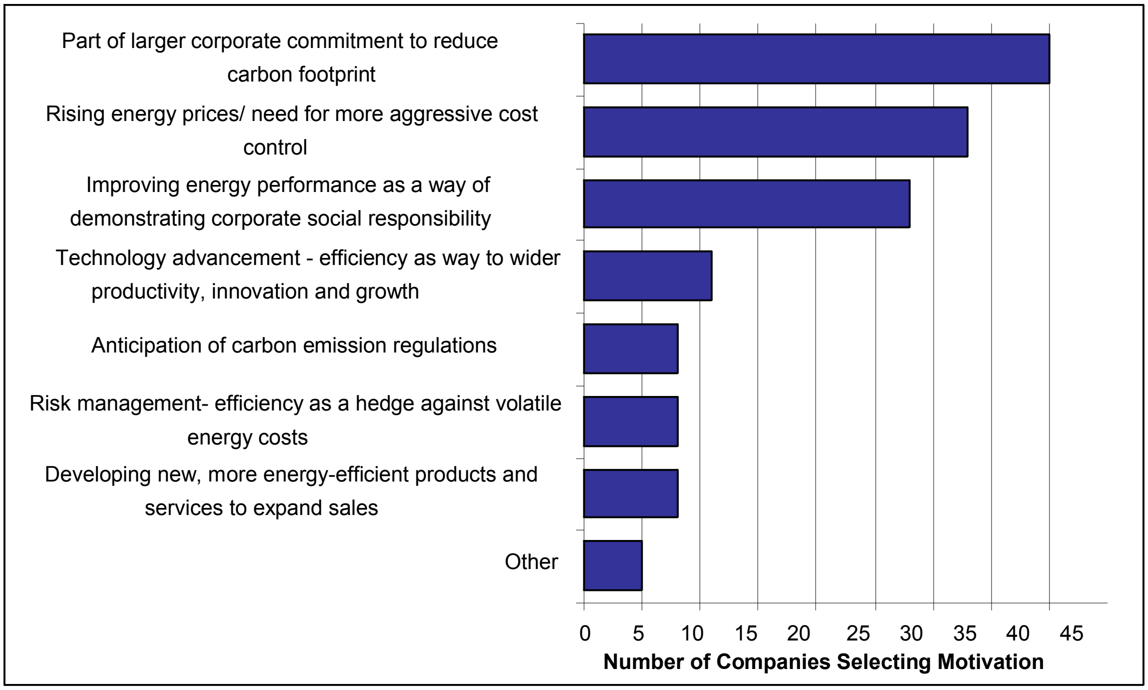


However, respondents varied considerably in the metrics they reported using for their energy savings targets. A simple percentage-of-energy savings target was the most commonly reported (21 respondents), where the goal was set in terms of reducing energy use by X% from Year A to Year B. Other respondents normalized their energy savings targets to a variety of metrics, including energy used per square foot of floor space, energy used per unit of product, or energy used per dollar of revenue. Some respondents set absolute savings targets, in energy units or in dollars.

Leading Motivations for Energy Efficiency Strategies

Respondents were asked to select the leading motivators for their energy efficiency strategies. Their answers are graphed in Figure 2. It is interesting to note that although the highest frequency of responses was that efficiency strategies are part of a corporate commitment to reduce the company's carbon footprint, the least-selected factor was anticipation of mandatory carbon emission regulations. This may reflect the sample's bias toward companies with an active voluntary commitment on climate issues. It may also reflect an understanding that most companies' facilities, except for larger power generation and industrial facilities, will not be directly regulated by carbon regulations, and that energy efficiency strategies have a sound business case with or without regulations, while also showing concrete action on reducing the company's carbon footprint.

Figure 2. Leading Motivations for Company Energy Efficiency Strategies



Scope, Staffing and Resourcing in Energy Management Strategies

Companies were asked whether their energy efficiency strategies are corporation-wide, or operate at the individual plant or division level. Almost all (94%) reported that the strategy operates corporation-wide; 3 respondents, or 6%, said that their efficiency strategies operate at a division level. However, in a follow-up question, 64% of respondents added that in addition to operating a corporation-wide strategy, they also quantify energy performance at the business unit or division level, and 81% quantify performance at the plant/facility level. Note that these percentages add up to more than 100 percent because respondents were able to select more than one business level at which they quantify their energy performance or energy savings.

Most respondents (60%) reported that they employ a full-time energy manager. Others reassigned existing staff or use other ways to support their efforts. Respondents also rated the relative level of effort, and the relative cost impact, of five basic elements of their efficiency strategy effort. Those rankings are summarized in Table 1. It is interesting to note that employee engagement ranks low in terms of management effort and dollar cost; later in the survey, many respondents noted how well their employees embraced their efficiency initiatives. This suggests that employee engagement strategies may become a larger part of companies' energy and climate strategies, especially in difficult economic times.

Table 1. Rankings of Key Program Elements by Level of Effort and Cost (5=greatest level of effort or cost)

Program Element	Labor Effort	Money Spent
Program management (data collection, reporting, project development, etc.)	5	2
Operations practices improvement (no cost to low cost)	4	3
Low-cost equipment measures (typically 1 year payback or less)	3	4
Larger capital projects (multi-year paybacks, capital financing, etc.)	2	5
Employee engagement communication, etc.	1	1

Leadership and Performance Accountability

Companies were asked which people or departments they considered to the most important champions for their efficiency strategies. CEOs and the senior management team were the most frequently selected choice, followed by plant/facility managers and operations staff. Environment/Health/Safety staff also were identified by many respondents. These results are illustrated in Table 2.

Table 2. Key Champions for Energy Efficiency Strategies

Champions	Number Selected
Board of Directors	3
CEO and Senior Management Team	37
Plant or Facility Managers	33
Accounting and Finance	4
Environmental Health and Safety	21
Operations	29
Strategic Planning	3
Other	12

Companies were also asked how energy performance is used as an element of job performance and career advancement. 49% said they explicitly include energy efficiency performance in annual review and compensation processes. We also asked which levels of management energy efficiency performance affected in this way; those results are shown in Table 3.

Table 3. Levels of Management Accountable for Energy Performance

	Number of Mentions	Percent*
Senior management ("C-level")	17	38%
Officer level (Vice Presidents/other officers)	24	53%
Corporate Energy Manager	26	58%
Middle management (Division/dept. managers)	27	60%
Facility level (Plant managers, facility mangers)	39	87%

* Percentages add up to more than 100 percent because respondents were able to select more than one business level at which energy performance is measured and accounted for.

Employee Engagement

Companies were asked whether employee engagement, beyond the core energy management leadership team, is a formal element of the corporate energy management strategy. 89% of respondents said yes, though a wide variety of employee engagement methods were reported. Those responses are summarized in Table 4.

Table 4. Methods Used for Employee Education and Engagement

Categories	Mentions	Percent of Respondents
Newsletters or Reports / E-mails / Bulk Communication	16	33%
Education and/or Trainings	11	23%
Developed a Green Program for Employees	9	19%
Green or Energy Teams / Committees	8	17%
Intranet or Website	8	17%
Employee Suggestion Box	7	15%
Energy Efficiency Campaigns or Initiatives	6	12%
Posting Signs or Posters	5	10%
Rewards / Incentive system	5	10%
Energy Themed Forums, Brownbag Lunches, Meetings and/or Conferences	5	10%
Surveys	2	4%

* Percentages add up to more than 100 percent because respondents were able to select more than one business level at which energy performance is measured and accounted for.

Finance and Risk Management Aspects of Energy Efficiency Investments

Respondents were asked whether they use a standard financial criterion to assess energy efficiency projects. 91% answered yes to this question; the distribution of responses showed that simple payback and internal rate of return were the most common criteria, though some respondents also used net operating income, lifecycle cost, and net present value methods.

15 companies reported the payback periods they use. All applied payback periods no longer than 5 years—3 years or less was the most commonly selected period. Payback periods responses are summarized in Table 5.

Table 5. Range of Reported Investment Payback Periods

Payback period	Number Selected
One Year	2
Two Years	4
Three Years	6
Four Years	1
Five years	2

Ten companies reported an IRR figure, as shown in Table 6. Half of these respondents used IRR criteria of 15% or less, and the highest reported was 35%.

Table 6. Range of Reported Investment Internal Rate of Return

IRR Threshold	Number Selected
10-15%	2
15%	3
18%	1
20%	1
22%	1
25%	1
35%	1

Beyond basic criteria like simple payback and IRR, we also asked companies if they employ any additional considerations or special processes for energy efficiency projects to ensure that efficiency projects get funded that would otherwise fail corporate financial criteria. 63% answered yes to this question. Within that group of 29, the following additional initiatives were mentioned:

- **Established a special pool of capital available only for energy efficiency projects.** 13 companies reported this approach, with capital pools ranging from $3 million--$240 million, available over a period of 1-7 years. The average capital pool was $51.3 million; on an annualized basis, the average pool was $12.8 million.
- **Build in assumptions about future energy price increases or supply shocks into the proposal to enhance financial or risk management benefits of efficiency projects.** 12 companies reported this practice, though no price information was provided.
- **Build in assumptions about future carbon prices to enhance benefits of efficiency projects.** Six reported their carbon price expectations. While these results are not statistically meaningful, these respondents expect carbon prices to exceed $30/ton by 2020.
- **Take into account the relative lack of risk involved in energy efficiency projects.** Ten companies reported this approach, though no specific metrics were provided.
- **Take into account co-benefits of improved energy efficiency.** All 29 selected at least one co-benefit of efficiency investments. Enhanced corporation reputation was the mostly frequently selected choice, followed by improved competitive positioning. Employee morale and productivity were also selected by many respondents.

- **Bundling multiple energy efficiency projects into one larger budget item.** 11 companies reported bundling efficiency projects into aggregated investments, partly to overcome the difficulty of gaining corporate level attention for relatively small expenditures.

Challenges in Mounting Internal Initiatives

Companies were asked to identify the biggest challenges in developing and sustaining efficiency initiatives for internal operations. Lack of funding was the most widely selected factor, followed by lack of staff time for project development, and organizational barriers.

Supply Chain Initiatives

Eight respondents (17% of total sample) reported having estimated suppliers' "energy footprint" or total usage. For those who had made such estimates, we asked whether the suppliers' footprint was smaller than, equal to, or larger than the company's internal operations energy footprint. One respondent said their suppliers' footprint was smaller, one equal, and five larger than their internal energy usage. This appears to be typical—most companies that estimate suppliers' footprint tend to find that their suppliers' energy usage (and often their carbon footprint) outweighs their own.

Respondents were also asked, independently of the footprint-measurement question, what energy efficiency measures they have undertaken with suppliers. The most common response was providing information on third-party efficiency programs or resources, followed by setting up energy/carbon reporting systems, providing technical assistance, and in a few cases, changing suppliers based on energy/carbon performance. Table 7 summarizes these responses.

Table 7. Energy Efficiency Measures Taken with Suppliers

Supplier Energy Efficiency Measure	Frequency	Percent*
Set up a measuring/reporting system for their energy/carbon performance	10	21%
Set specific energy or GHG reduction targets	0	0%
Provided information on energy efficiency programs and other resources available from21 third-party sources		44%
Provided technical services (at your cost) to improve their energy/carbon performance	8	17%
Changed suppliers based on identification suppliers with superior energy/carbon effici performance	6	12%
Other initiatives	10	21%

* Percentages add up to more than 100 percent because respondents were able to select more than one energy efficiency strategy that they have undertaken with their suppliers.

Companies were also asked what the biggest challenges were in developing and sustaining efficiency initiatives in the supply chain. Getting suppliers' data was the most frequently selected factor, followed by cost issues and supplier resistance.

Products and Services

Companies reported having taken various initiatives with their products and services, at a rather high rate. 55% (26) had calculated the energy footprint from their products and services. On a comparative basis, 7 reported their product/service footprint to be smaller than their internal operations, 2 reported they were equal, and 17 reported product/service footprints larger than internal operations.

Somewhat surprisingly, 81% (38) reported that they had modified their products and services to enhance or offer new levels of energy efficiency performance. When asked to identify their motives for doing this, companies reported a range of motives: of these, the most frequently selected were, "Take advantage of new market trends brought on by consumer concerns about energy prices", "Take advantage of new market trends brought on by consumer concerns about environmental issues", and "Respond to competitive pressures".

Respondents were asked to identify the biggest challenges they faced in developing, rolling out, or sustaining sales of energy efficiency products or services. The most frequent responses were cost barriers, customer unwillingness to pay, and engineering barriers.

Lessons Learned, Remaining Challenges, and Future Needs

The last section of the survey asked companies to sum up the successes, setbacks, lessons, and future needs they see for their energy efficiency strategies. The biggest successes observed in companies energy efficiency strategies included the following (top five most frequent responses shown):

- Meeting / Exceeding Goals 48% of respondents
- Implementing Corporate Wide Plan 23%
- Increasing Employee Involvement 21%
- Formalizing a Policy / Strategy 15%
- Implementing at Local Level 15%

Almost half of respondents reported meeting their goals. Many setbacks were also reported, including:

- Limited Capital for EE 19%
- Limited Leadership Buy-In 10%
- Improving EE is Harder than Expected 10%
- Competing Priorities / Resources 6%
- Lagging Momentum / Employee Interest 6%

Companies reported the most successful corrective actions they took in response to these setbacks, summarized as follows:

- Doing Audits for EE improvements 12%
- Revising a Strategy / Goals 10%
- Building Teams to Support the Effort 10%

- Increasing Employee Involvement 8%
- Developing Feedback Mechanisms 8%

We also asked companies to report any surprises or unexpected results that they experienced. Several companies reported on this, with the following summary of responses:

- Employee Interest/Involvement 15%
- Immediacy of Meeting Goals/Success 15%
- Difficulty in Implementing Strategies 6%
- Wealth of Ideas / Opportunities 6%
- Difficulty in Finding Resources 4%

Respondents were asked to report the most important lessons learned since implementing their energy efficiency strategy. Responses are summarized in Table 8. The most frequently reported lesson was the need for better communication and coordination among units of the company, followed by the need to gain support from leadership, the need to actively engage employees, and the need for measurement and feedback in sustaining success.

Table 8. Key Lessons Learned in Implementing Efficiency Strategies

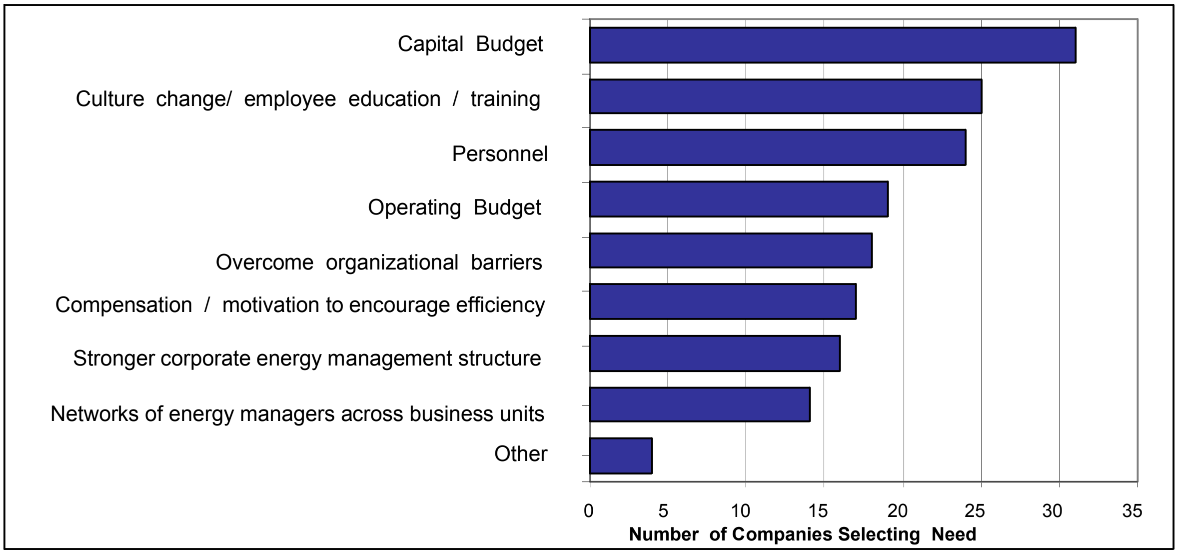
Categories	Frequency	Percent
Better Communication/Coordination Between Units	10	20.8%
Support from Management / Leadership Buy-In	7	14.6%
Employee Interest/Involvement in Energy Policy	6	12.5%
Developing a Feedback Mechanism / Measuring Results	6	12.5%
Need for Funding / Lack of Capital	4	8.3%
Setting Clear, Realistic Goals	3	6.3%
Continuous focus/awareness	2	4.2%
Other	22	45.8%

Companies reported the largest ongoing challenges keeping them from realizing the company's energy management goals. Need for capital to pay for projects was the greatest single ongoing challenge, outnumbering any other single item by a four-to-one ratio.

The final questions respondents were asked probed their most pressing needs to sustain and improve their energy management efforts, both for specific efficiency improvements and in terms of corporate-wide resources. As was shown in earlier responses on challenges, financial resources head the list of respondents' needs for specific efficiency improvements, followed by better management tools and technical information and assistance.

Looking more broadly at corporation-wide needs, respondents still saw capital needs as paramount. However, at the corporate level, culture change/education/training was tied with personnel needs for second place, followed by increased operating budget support, reducing organizational barriers, and better compensation and motivation systems to encourage efficiency. Figure 3 summarize these responses.

Figure 3. Ongoing Needs to Support Corporation-Wide Efficiency Strategies



Summary of Findings

The Pew Center survey brings to light several interesting facts and trends in corporate energy management, and helps identify key attributes on energy efficiency shared by leading large companies. Key findings include:

- Almost half of respondents reported setting quantified energy savings goals: the average was 20% of base year energy usage over nine years, or an annualized savings target of 2.2%
- 60% had full-time energy managers, 87% made facility/plant managers accountable for energy performance; 38% set energy performance goals for senior management.
- Over 90% of respondents reported standardized financial criteria: simple payback and IRR were the most frequent. Most simple payback thresholds were three years or less; most IRR thresholds were 15% or more.
- Most companies used other ways to support efficiency investment, including dedicated pools of capital, accounting for future energy and carbon prices, and estimating co-benefits.
- Less than half of respondents had taken specific actions to encourage energy efficiency in their supply chains; some had estimated their suppliers' energy/carbon footprint, and others established metrics and reporting systems to measure supplier performance.
- A surprisingly high 81% of respondents had modified their products and services to increase their energy efficiency; 55% had measured the energy footprint of their products and services.
- Among the surprises companies reported, the most common was the enthusiastic response they got from engaging employees.
- The greatest ongoing needs reported were greater capital and operating budgets, change in company culture/employee engagement, more personnel resources, and reduction of internal barriers to energy efficiency investment.

Conclusions

This survey sheds new light on emerging trends in energy management at some of the largest and most progressive companies. While the survey was deliberately aimed at companies known to be active in the energy efficiency and climate policy field, it produced responses that help articulate the key elements of success in corporate energy management. These include:

1. Efficiency as an integral part of corporate strategic planning and risk assessment
2. Real and sustainable senior management leadership and organizational support
3. Specific, aggressive, measurable, and accountable energy efficiency goals
4. A robust tracking and performance measurement system
5. Commitment of organizational resources in a substantial and sustained way
6. Documentation of results with quantitative, company-wide data
7. Communication of results both internally and externally

The survey produced some surprising findings, including the importance of employee engagement and enthusiasm. While efficiency has often been a behind-the-scenes engineering function driven by technology investment, today's most successful efforts draw as much on human capital and culture change to drive results as they do engineering expertise and technology investment.

Next steps in the research process include development of the case studies, which are expected to provide additional depth and detail to some of the key findings identified through the survey. For example, the case studies will seek to describe exactly how selected companies set efficiency targets and measure progress toward their goals. The case studies will also explore company experiences with various financing mechanisms for efficiency projects, including the use of dedicated pools of capital, and budgeting techniques such as bundling multiple small projects together into one larger fiscal item. Ultimately, the aim of the report is to integrate survey and case study findings to provide a comprehensive set of tools and resources for companies seeking to enhance their energy efficiency efforts.

The Pew Center also intends to develop a separate section of its Web site devoted to the topic of corporate energy efficiency. It plans to develop more case studies and additional resources that capture the advancing state of the art on this fast moving issue.

BEFORE THE

PUBLIC UTILITY COMMISSION OF OREGON

NW Natural

Exhibit 504 of Barbara Summers

UM 1744
Carbon Emission Reduction Program
Combined Heat & Power (CHP)

WSU Model with 2012 eGrid
Non-baseload Emissions

October 16, 2015

2010 eGrid Lines 5:394
2012 eGrid Lines 397:794

	2010 eGrid								
Scenario	**NWN CO2e Reduction Incentive ($/tonne/yr)**	**Carbon Emission Reduction**	**ETO Grant Rate ($/kWh)**	**ETO Grant Amount**	**CapEx, $ per kW**	**Before-Tax Simple Payback**	**Net Present Value**	**Project IRR**	**After-Tax Discounted Payback**
Hospital - 800,000 sf with Two 800 kW Recip Engines, eGRID non-baseload baseline	$0	3,249	$ 0.08	317,834		8.9	$ (388,404)	4.9%	Exceeds Project Life
	$30	3,249	$ 0.08	317,834		5.3	$ 270,344	13.6%	9.0
	$40	3,249	$ 0.08	317,834		4.7	$ 489,927	16.6%	7.1
	$50	3,249	$ 0.08	317,834		4.2	$ 709,510	19.5%	5.9
	$60	3,249	$ 0.08	317,834		3.8	$ 929,093	22.5%	5.2
	$70	3,249	$ 0.08	317,834		3.5	$ 1,148,676	25.6%	4.7
	$80	3,249	$ 0.08	317,834		3.2	$ 1,368,259	28.6%	4.4
	$30	2,144	$ 0.08	317,834		6.2	$ 46,370	10.6%	13.6
	$40	2,144	$ 0.08	317,834		5.6	$ 191,295	12.6%	9.9
	$50	2,144	$ 0.08	317,834		5.1	$ 336,219	14.5%	8.3
	$60	2,144	$ 0.08	317,834		4.7	$ 481,144	16.5%	7.2
	$70	2,144	$ 0.08	317,834		4.4	$ 626,069	18.4%	6.3
	$80	2,144	$ 0.08	317,834		4.1	$ 770,994	20.4%	5.7
	$30	1,072	$ 0.08	317,834		7.3	$ (171,017)	7.7%	Exceeds Project Life
	$40	1,072	$ 0.08	317,834		6.9	$ (98,555)	8.7%	Exceeds Project Life
	$50	1,072	$ 0.08	317,834		6.5	$ (26,093)	9.7%	Exceeds Project Life
	$60	1,072	$ 0.08	317,834		6.2	$ 46,370	10.6%	13.6
	$70	1,072	$ 0.08	317,834		5.9	$ 118,832	11.6%	11.5
	$80	1,072	$ 0.08	317,834		5.6	$ 191,295	12.6%	9.9
	$0	3,249	$ 0.25	500,000		7.6	$ (229,608)	6.8%	Exceeds Project Life
	$30	3,249	$ 0.25	500,000		4.6	$ 429,141	16.0%	7.3
	$40	3,249	$ 0.25	500,000		4.0	$ 648,724	19.1%	5.9
	$50	3,249	$ 0.25	500,000		3.6	$ 868,306	22.2%	5.1
	$60	3,249	$ 0.25	500,000		3.3	$ 1,087,889	25.3%	4.6
	$70	3,249	$ 0.25	500,000		3.0	$ 1,307,472	28.5%	4.3
	$80	3,249	$ 0.25	500,000		2.7	$ 1,527,055	31.7%	4.0
	$30	2,144	$ 0.25	500,000		5.3	$ 205,166	12.9%	9.7
	$40	2,144	$ 0.25	500,000		4.8	$ 350,091	14.9%	8.0
	$50	2,144	$ 0.25	500,000		4.4	$ 495,015	16.9%	6.8
	$60	2,144	$ 0.25	500,000		4.1	$ 639,940	19.0%	5.9
	$70	2,144	$ 0.25	500,000		3.8	$ 784,865	21.0%	5.4
	$80	2,144	$ 0.25	500,000		3.5	$ 929,790	23.1%	4.9
	$30	1,072	$ 0.25	500,000		6.3	$ (12,221)	9.8%	Exceeds Project Life
	$40	1,072	$ 0.25	500,000		5.9	$ 60,241	10.8%	13.1
	$50	1,072	$ 0.25	500,000		5.6	$ 132,704	11.9%	11.2
	$60	1,072	$ 0.25	500,000		5.3	$ 205,166	12.9%	9.7
	$70	1,072	$ 0.25	500,000		5.0	$ 277,628	13.9%	8.7
	$80	1,072	$ 0.25	500,000		4.8	$ 350,091	14.9%	8.0
	$0	1,297	$ 0.08	110,183		8.7	$ (119,706)	5.2%	Exceeds Project Life
	$30	1,297	$ 0.08	110,183		4.8	$ 143,298	15.9%	7.5
	$40	1,297	$ 0.08	110,183		4.2	$ 230,965	19.5%	5.9
	$50	1,297	$ 0.08	110,183		3.7	$ 318,633	23.1%	5.0
	$60	1,297	$ 0.08	110,183		3.3	$ 406,301	26.8%	4.6
	$70	1,297	$ 0.08	110,183		3.0	$ 493,969	30.6%	4.2
	$80	1,297	$ 0.08	110,183		2.8	$ 581,637	34.4%	3.9
	$30	856	$ 0.08	110,183		5.7	$ 53,876	12.2%	10.5
	$40	856	$ 0.08	110,183		5.1	$ 111,737	14.6%	8.3
	$50	856	$ 0.08	110,183		4.6	$ 169,598	16.9%	6.9
	$60	856	$ 0.08	110,183		4.2	$ 227,459	19.3%	6.0
	$70	856	$ 0.08	110,183		3.9	$ 285,320	21.7%	5.3
	$80	856	$ 0.08	110,183		3.6	$ 343,180	24.2%	4.9
	$30	428	$ 0.08	110,183		6.9	$ (32,915)	8.7%	Exceeds Project Life
	$40	428	$ 0.08	110,183		6.4	$ (3,984)	9.8%	Exceeds Project Life

Reciprocating Engine - 500 kW, eGRID non-baseload baseline	$50	428	$	0.08	110,183		6.0	$	24,946	11.0%	12.7
	$60	428	$	0.08	110,183		5.7	$	53,876	12.2%	10.5
	$70	428	$	0.08	110,183		5.4	$	82,807	13.4%	9.2
	$80	428	$	0.08	110,183		5.1	$	111,737	14.6%	8.3
	$0	1,297	$	0.25	344,323		3.9	$	84,397	14.2%	8.4
	$30	1,297	$	0.25	344,323		2.1	$	347,400	27.0%	4.0
	$40	1,297	$	0.25	344,323		1.9	$	435,068	31.3%	3.8
	$50	1,297	$	0.25	344,323		1.6	$	522,736	35.7%	3.5
	$60	1,297	$	0.25	344,323		1.5	$	610,404	40.1%	3.3
	$70	1,297	$	0.25	344,323		1.3	$	698,071	44.6%	3.1
	$80	1,297	$	0.25	344,323		1.2	$	785,739	49.1%	3.0
	$30	856	$	0.25	344,323		2.5	$	257,979	22.7%	4.6
	$40	856	$	0.25	344,323		2.2	$	315,840	25.5%	4.2
	$50	856	$	0.25	344,323		2.0	$	373,700	28.3%	3.9
	$60	856	$	0.25	344,323		1.9	$	431,561	31.2%	3.8
	$70	856	$	0.25	344,323		1.7	$	489,422	34.0%	3.6
	$80	856	$	0.25	344,323		1.6	$	547,283	36.9%	3.5
	$30	428	$	0.25	344,323		3.0	$	171,188	18.4%	5.4
	$40	428	$	0.25	344,323		2.8	$	200,118	19.8%	5.0
	$50	428	$	0.25	344,323		2.7	$	229,048	21.3%	4.7
	$60	428	$	0.25	344,323		2.5	$	257,979	22.7%	4.6
	$70	428	$	0.25	344,323		2.4	$	286,909	24.1%	4.4
	$80	428	$	0.25	344,323		2.2	$	315,840	25.5%	4.2
Reciprocating Engine - 4.3 MW, eGRID non-baseload baseline	$0	15,051	$	0.08	500,000		3.9	$	1,881,269	18.7%	7.1
	$30	15,051	$	0.08	500,000		2.6	$	4,933,154	34.6%	3.9
	$40	15,051	$	0.08	500,000		2.3	$	5,950,449	40.5%	3.6
	$50	15,051	$	0.08	500,000		2.1	$	6,967,744	46.6%	3.2
	$60	15,051	$	0.08	500,000		1.9	$	7,985,039	53.1%	3.0
	$70	15,051	$	0.08	500,000		1.8	$	9,002,335	60.0%	2.7
	$80	15,051	$	0.08	500,000		1.6	$	10,019,630	67.4%	2.6
	$30	9,934	$	0.08	500,000		2.9	$	3,895,513	28.9%	4.5
	$40	9,934	$	0.08	500,000		2.7	$	4,566,928	32.6%	4.1
	$50	9,934	$	0.08	500,000		2.5	$	5,238,343	36.3%	3.8
	$60	9,934	$	0.08	500,000		2.3	$	5,909,758	40.2%	3.6
	$70	9,934	$	0.08	500,000		2.2	$	6,581,172	44.2%	3.4
	$80	9,934	$	0.08	500,000		2.1	$	7,252,587	48.4%	3.2
	$30	4,967	$	0.08	500,000		3.4	$	2,888,391	23.7%	5.3
	$40	4,967	$	0.08	500,000		3.2	$	3,224,099	25.4%	4.9
	$50	4,967	$	0.08	500,000		3.1	$	3,559,806	27.1%	4.7
	$60	4,967	$	0.08	500,000		2.9	$	3,895,513	28.9%	4.5
	$70	4,967	$	0.08	500,000		2.8	$	4,231,221	30.7%	4.3
	$80	4,967	$	0.08	500,000		2.7	$	4,566,928	32.6%	4.1
	$0	15,051	$	0.25	500,000		3.9	$	1,881,269	18.7%	7.1
	$30	15,051	$	0.25	500,000		2.6	$	4,933,154	34.6%	3.9
	$40	15,051	$	0.25	500,000		2.3	$	5,950,449	40.5%	3.6
	$50	15,051	$	0.25	500,000		2.1	$	6,967,744	46.6%	3.2
	$60	15,051	$	0.25	500,000		1.9	$	7,985,039	53.1%	3.0
	$70	15,051	$	0.25	500,000		1.8	$	9,002,335	60.0%	2.7
	$80	15,051	$	0.25	500,000		1.6	$	10,019,630	67.4%	2.6
	$30	9,934	$	0.25	500,000		2.9	$	3,895,513	28.9%	4.5
	$40	9,934	$	0.25	500,000		2.7	$	4,566,928	32.6%	4.1
	$50	9,934	$	0.25	500,000		2.5	$	5,238,343	36.3%	3.8
	$60	9,934	$	0.25	500,000		2.3	$	5,909,758	40.2%	3.6
	$70	9,934	$	0.25	500,000		2.2	$	6,581,172	44.2%	3.4
	$80	9,934	$	0.25	500,000		2.1	$	7,252,587	48.4%	3.2
	$30	4,967	$	0.25	500,000		3.4	$	2,888,391	23.7%	5.3
	$40	4,967	$	0.25	500,000		3.2	$	3,224,099	25.4%	4.9
	$50	4,967	$	0.25	500,000		3.1	$	3,559,806	27.1%	4.7
	$60	4,967	$	0.25	500,000		2.9	$	3,895,513	28.9%	4.5
	$70	4,967	$	0.25	500,000		2.8	$	4,231,221	30.7%	4.3
	$80	4,967	$	0.25	500,000		2.7	$	4,566,928	32.6%	4.1
	$0	62,652	$	0.08	500,000		5.4	$	3,935,982	13.7%	10.9
	$30	62,652	$	0.08	500,000		3.7	$	16,639,978	27.2%	5.2
	$40	62,652	$	0.08	500,000		3.3	$	20,874,644	32.2%	4.5
	$50	62,652	$	0.08	500,000		3.0	$	25,109,309	37.5%	3.9

	$60	62,652	$	0.08	500,000		2.8	$	29,343,975	43.2%	3.5
	$70	62,652	$	0.08	500,000		2.6	$	33,578,640	49.2%	3.2
	$80	62,652	$	0.08	500,000		2.4	$	37,813,305	55.7%	2.9
	$30	41,350	$	0.08	500,000		4.1	$	12,320,620	22.3%	6.3
	$40	41,350	$	0.08	500,000		3.8	$	15,115,499	25.4%	5.5
	$50	41,350	$	0.08	500,000		3.6	$	17,910,378	28.6%	4.9
	$60	41,350	$	0.08	500,000		3.3	$	20,705,257	32.0%	4.5
	$70	41,350	$	0.08	500,000		3.2	$	23,500,136	35.5%	4.1
	$80	41,350	$	0.08	500,000		3.0	$	26,295,016	39.1%	3.8
	$30	20,675	$	0.08	500,000		4.7	$	8,128,301	17.8%	8.0
	$40	20,675	$	0.08	500,000		4.5	$	9,525,741	19.3%	7.3
	$50	20,675	$	0.08	500,000		4.3	$	10,923,180	20.8%	6.7
	$60	20,675	$	0.08	500,000		4.1	$	12,320,620	22.3%	6.3
Gas Turbine - 21.7 MW, eGRID non-baseload baseline, Without Gas Compression	$70	20,675	$	0.08	500,000		4.0	$	13,718,059	23.8%	5.8
	$80	20,675	$	0.08	500,000		3.8	$	15,115,499	25.4%	5.5
	$0	62,652	$	0.25	500,000		5.4	$	3,935,982	13.7%	10.9
	$30	62,652	$	0.25	500,000		3.7	$	16,639,978	27.2%	5.2
	$40	62,652	$	0.25	500,000		3.3	$	20,874,644	32.2%	4.5
	$50	62,652	$	0.25	500,000		3.0	$	25,109,309	37.5%	3.9
	$60	62,652	$	0.25	500,000		2.8	$	29,343,975	43.2%	3.5
	$70	62,652	$	0.25	500,000		2.6	$	33,578,640	49.2%	3.2
	$80	62,652	$	0.25	500,000		2.4	$	37,813,305	55.7%	2.9
	$30	41,350	$	0.25	500,000		4.1	$	12,320,620	22.3%	6.3
	$40	41,350	$	0.25	500,000		3.8	$	15,115,499	25.4%	5.5
	$50	41,350	$	0.25	500,000		3.6	$	17,910,378	28.6%	4.9
	$60	41,350	$	0.25	500,000		3.3	$	20,705,257	32.0%	4.5
	$70	41,350	$	0.25	500,000		3.2	$	23,500,136	35.5%	4.1
	$80	41,350	$	0.25	500,000		3.0	$	26,295,016	39.1%	3.8
	$30	20,675	$	0.25	500,000		4.7	$	8,128,301	17.8%	8.0
	$40	20,675	$	0.25	500,000		4.5	$	9,525,741	19.3%	7.3
	$50	20,675	$	0.25	500,000		4.3	$	10,923,180	20.8%	6.7
	$60	20,675	$	0.25	500,000		4.1	$	12,320,620	22.3%	6.3
	$70	20,675	$	0.25	500,000		4.0	$	13,718,059	23.8%	5.8
	$80	20,675	$	0.25	500,000		3.8	$	15,115,499	25.4%	5.5
	$0	132,175	$	0.08	500,000		5.8	$	4,464,116	11.9%	12.6
	$30	132,175	$	0.08	500,000		3.9	$	31,265,330	24.9%	5.7
	$40	132,175	$	0.08	500,000		3.6	$	40,199,068	29.8%	4.9
	$50	132,175	$	0.08	500,000		3.3	$	49,132,807	35.1%	4.3
	$60	132,175	$	0.08	500,000		3.0	$	58,066,545	40.6%	3.8
	$70	132,175	$	0.08	500,000		2.8	$	67,000,283	46.5%	3.4
	$80	132,175	$	0.08	500,000		2.6	$	75,934,021	52.9%	3.1
	$30	87,235	$	0.08	500,000		4.4	$	22,152,917	20.2%	7.0
	$40	87,235	$	0.08	500,000		4.1	$	28,049,184	23.2%	6.1
	$50	87,235	$	0.08	500,000		3.8	$	33,945,452	26.4%	5.4
	$60	87,235	$	0.08	500,000		3.6	$	39,841,719	29.6%	4.9
	$70	87,235	$	0.08	500,000		3.4	$	45,737,986	33.0%	4.5
	$80	87,235	$	0.08	500,000		3.2	$	51,634,253	36.6%	4.1
	$30	43,618	$	0.08	500,000		5.0	$	13,308,516	15.9%	9.0
	$40	43,618	$	0.08	500,000		4.8	$	16,256,650	17.3%	8.2
	$50	43,618	$	0.08	500,000		4.6	$	19,204,784	18.8%	7.6
Gas Turbine - 45 MW, eGRID non-baseload baseline -- Without Gas Compression	$60	43,618	$	0.08	500,000		4.4	$	22,152,917	20.2%	7.0
	$70	43,618	$	0.08	500,000		4.3	$	25,101,051	21.7%	6.5
	$80	43,618	$	0.08	500,000		4.1	$	28,049,184	23.2%	6.1
	$0	132,175	$	0.25	500,000		5.8	$	4,464,116	11.9%	12.6
	$30	132,175	$	0.25	500,000		3.9	$	31,265,330	24.9%	5.7
	$40	132,175	$	0.25	500,000		3.6	$	40,199,068	29.8%	4.9
	$50	132,175	$	0.25	500,000		3.3	$	49,132,807	35.1%	4.3
	$60	132,175	$	0.25	500,000		3.0	$	58,066,545	40.6%	3.8
	$70	132,175	$	0.25	500,000		2.8	$	67,000,283	46.5%	3.4
	$80	132,175	$	0.25	500,000		2.6	$	75,934,021	52.9%	3.1
	$30	87,235	$	0.25	500,000		4.4	$	22,152,917	20.2%	7.0
	$40	87,235	$	0.25	500,000		4.1	$	28,049,184	23.2%	6.1
	$50	87,235	$	0.25	500,000		3.8	$	33,945,452	26.4%	5.4
	$60	87,235	$	0.25	500,000		3.6	$	39,841,719	29.6%	4.9
	$70	87,235	$	0.25	500,000		3.4	$	45,737,986	33.0%	4.5

	$80	87,235	$ 0.25	500,000		3.2	$ 51,634,253	36.6%	4.1
	$30	43,618	$ 0.25	500,000		5.0	$ 13,308,516	15.9%	9.0
	$40	43,618	$ 0.25	500,000		4.8	$ 16,256,650	17.3%	8.2
	$50	43,618	$ 0.25	500,000		4.6	$ 19,204,784	18.8%	7.6
	$60	43,618	$ 0.25	500,000		4.4	$ 22,152,917	20.2%	7.0
	$70	43,618	$ 0.25	500,000		4.3	$ 25,101,051	21.7%	6.5
	$80	43,618	$ 0.25	500,000		4.1	$ 28,049,184	23.2%	6.1
Gas Turbine - 45 MW, eGRID (2010) non-baseload baseline, 70% CapEx -- With Gas Compression	$0	132,175	$ 0.08	$500,000	$905	4.0	$ 17,735,773	20.6%	7.3
	$15	132,175	$ 0.08	$500,000	$905	3.2	$ 31,136,381	30.3%	4.9
	$30	132,175	$ 0.08	$500,000	$905	2.7	$ 44,536,988	41.6%	3.8
	$45	132,175	$ 0.08	$500,000	$905	2.3	$ 57,937,595	54.7%	3.0
	$60	132,175	$ 0.08	$500,000	$905	2.0	$ 71,338,202	70.2%	2.6
	$75	132,175	$ 0.08	$500,000	$905	1.8	$ 84,738,810	88.7%	2.2
	$90	132,175	$ 0.08	$500,000	$905	1.6	$ 98,139,417	111.4%	2.0
	$105	132,175	$ 0.08	$500,000	$905	1.5	$ 111,540,024	140.1%	1.8
	$120	132,175	$ 0.08	$500,000	$905	1.4	$ 124,940,631	177.4%	1.6
	$135	132,175	$ 0.08	$500,000	$905	1.3	$ 138,341,239	228.1%	1.5
	$150	132,175	$ 0.08	$500,000	$905	1.2	$ 151,741,846	301.1%	1.4
	$0	87,235	$ 0.08	$500,000	$905	4.0	$ 17,735,773	20.6%	7.3
	$15	87,235	$ 0.08	$500,000	$905	3.4	$ 26,580,174	26.9%	5.5
	$30	87,235	$ 0.08	$500,000	$905	3.0	$ 35,424,575	33.8%	4.4
	$45	87,235	$ 0.08	$500,000	$905	2.7	$ 44,268,976	41.4%	3.8
	$60	87,235	$ 0.08	$500,000	$905	2.4	$ 53,113,377	49.8%	3.3
	$75	87,235	$ 0.08	$500,000	$905	2.2	$ 61,957,777	59.1%	2.9
	$90	87,235	$ 0.08	$500,000	$905	2.0	$ 70,802,178	69.5%	2.6
	$105	87,235	$ 0.08	$500,000	$905	1.9	$ 79,646,579	81.2%	2.3
	$120	87,235	$ 0.08	$500,000	$905	1.8	$ 88,490,980	94.6%	2.1
	$135	87,235	$ 0.08	$500,000	$905	1.6	$ 97,335,380	109.9%	2.0
	$150	87,235	$ 0.08	$500,000	$905	1.5	$ 106,179,781	127.8%	1.8
	$0	43,618	$ 0.08	$500,000	$905	4.0	$ 17,735,773	20.6%	7.3
	$15	43,618	$ 0.08	$500,000	$905	3.7	$ 22,157,974	23.7%	6.2
	$30	43,618	$ 0.08	$500,000	$905	3.4	$ 26,580,174	26.9%	5.5
	$45	43,618	$ 0.08	$500,000	$905	3.2	$ 31,002,375	30.2%	4.9
	$60	43,618	$ 0.08	$500,000	$905	3.0	$ 35,424,575	33.8%	4.4
	$75	43,618	$ 0.08	$500,000	$905	2.9	$ 39,846,775	37.5%	4.1
	$90	43,618	$ 0.08	$500,000	$905	2.7	$ 44,268,976	41.4%	3.8
	$105	43,618	$ 0.08	$500,000	$905	2.6	$ 48,691,176	45.5%	3.5
	$120	43,618	$ 0.08	$500,000	$905	2.4	$ 53,113,377	49.8%	3.3
	$135	43,618	$ 0.08	$500,000	$905	2.3	$ 57,535,577	54.3%	3.1
	$150	43,618	$ 0.08	$500,000	$905	2.2	$ 61,957,777	59.1%	2.9
	$0	132,175	$ 0.25	$500,000	$905	4.0	$ 17,735,773	20.6%	7.3
	$15	132,175	$ 0.25	$500,000	$905	3.2	$ 31,136,381	30.3%	4.9
	$30	132,175	$ 0.25	$500,000	$905	2.7	$ 44,536,988	41.6%	3.8
	$45	132,175	$ 0.25	$500,000	$905	2.3	$ 57,937,595	54.7%	3.0
	$60	132,175	$ 0.25	$500,000	$905	2.0	$ 71,338,202	70.2%	2.6
	$75	132,175	$ 0.25	$500,000	$905	1.8	$ 84,738,810	88.7%	2.2
	$90	132,175	$ 0.25	$500,000	$905	1.6	$ 98,139,417	111.4%	2.0
	$105	132,175	$ 0.25	$500,000	$905	1.5	$ 111,540,024	140.1%	1.8
	$120	132,175	$ 0.25	$500,000	$905	1.4	$ 124,940,631	177.4%	1.6
	$135	132,175	$ 0.25	$500,000	$905	1.3	$ 138,341,239	228.1%	1.5
	$150	132,175	$ 0.25	$500,000	$905	1.2	$ 151,741,846	301.1%	1.4
	$0	87,235	$ 0.25	$500,000	$905	4.0	$ 17,735,773	20.6%	7.3
	$15	87,235	$ 0.25	$500,000	$905	3.4	$ 26,580,174	26.9%	5.5
	$30	87,235	$ 0.25	$500,000	$905	3.0	$ 35,424,575	33.8%	4.4
	$45	87,235	$ 0.25	$500,000	$905	2.7	$ 44,268,976	41.4%	3.8
	$60	87,235	$ 0.25	$500,000	$905	2.4	$ 53,113,377	49.8%	3.3
	$75	87,235	$ 0.25	$500,000	$905	2.2	$ 61,957,777	59.1%	2.9
	$90	87,235	$ 0.25	$500,000	$905	2.0	$ 70,802,178	69.5%	2.6
	$105	87,235	$ 0.25	$500,000	$905	1.9	$ 79,646,579	81.2%	2.3
	$120	87,235	$ 0.25	$500,000	$905	1.8	$ 88,490,980	94.6%	2.1
	$135	87,235	$ 0.25	$500,000	$905	1.6	$ 97,335,380	109.9%	2.0
	$150	87,235	$ 0.25	$500,000	$905	1.5	$ 106,179,781	127.8%	1.8
	$0	43,618	$ 0.25	$500,000	$905	4.0	$ 17,735,773	20.6%	7.3
	$15	43,618	$ 0.25	$500,000	$905	3.7	$ 22,157,974	23.7%	6.2
	$30	43,618	$ 0.25	$500,000	$905	3.4	$ 26,580,174	26.9%	5.5

	$45	43,618	$ 0.25	$500,000	$905	3.2	$ 31,002,375	30.2%	4.9
	$60	43,618	$ 0.25	$500,000	$905	3.0	$ 35,424,575	33.8%	4.4
	$75	43,618	$ 0.25	$500,000	$905	2.9	$ 39,846,775	37.5%	4.1
	$90	43,618	$ 0.25	$500,000	$905	2.7	$ 44,268,976	41.4%	3.8
	$105	43,618	$ 0.25	$500,000	$905	2.6	$ 48,691,176	45.5%	3.5
	$120	43,618	$ 0.25	$500,000	$905	2.4	$ 53,113,377	49.8%	3.3
	$135	43,618	$ 0.25	$500,000	$905	2.3	$ 57,535,577	54.3%	3.1
	$150	43,618	$ 0.25	$500,000	$905	2.2	$ 61,957,777	59.1%	2.9
Gas Turbine - 21.7 MW, eGRID (2010) non-baseload baseline -- With Gas Compression	$0	62,652	$ 0.08	$500,000	$1,413	5.7	$ 2,997,518	12.7%	11.8
	$15	62,652	$ 0.08	$500,000	$1,413	4.6	$ 9,349,517	18.8%	7.5
	$30	62,652	$ 0.08	$500,000	$1,413	3.9	$ 15,701,515	25.5%	5.5
	$45	62,652	$ 0.08	$500,000	$1,413	3.3	$ 22,053,513	32.7%	4.4
	$60	62,652	$ 0.08	$500,000	$1,413	2.9	$ 28,405,511	40.5%	3.7
	$75	62,652	$ 0.08	$500,000	$1,413	2.6	$ 34,757,509	49.1%	3.2
	$90	62,652	$ 0.08	$500,000	$1,413	2.4	$ 41,109,507	58.5%	2.8
	$105	62,652	$ 0.08	$500,000	$1,413	2.2	$ 47,461,505	68.9%	2.5
	$120	62,652	$ 0.08	$500,000	$1,413	2.0	$ 53,813,503	80.5%	2.3
	$135	62,652	$ 0.08	$500,000	$1,413	1.8	$ 60,165,501	93.6%	2.1
	$150	62,652	$ 0.08	$500,000	$1,413	1.7	$ 66,517,499	108.5%	1.9
	$0	41,350	$ 0.08	$500,000	$1,413	5.7	$ 2,997,518	12.7%	11.8
	$15	41,350	$ 0.08	$500,000	$1,413	4.9	$ 7,189,837	16.7%	8.6
	$30	41,350	$ 0.08	$500,000	$1,413	4.3	$ 11,382,156	20.9%	6.7
	$45	41,350	$ 0.08	$500,000	$1,413	3.9	$ 15,574,475	25.3%	5.5
	$60	41,350	$ 0.08	$500,000	$1,413	3.5	$ 19,766,793	30.0%	4.7
	$75	41,350	$ 0.08	$500,000	$1,413	3.2	$ 23,959,112	35.0%	4.2
	$90	41,350	$ 0.08	$500,000	$1,413	3.0	$ 28,151,431	40.2%	3.8
	$105	41,350	$ 0.08	$500,000	$1,413	2.7	$ 32,343,750	45.8%	3.4
	$120	41,350	$ 0.08	$500,000	$1,413	2.6	$ 36,536,068	51.6%	3.1
	$135	41,350	$ 0.08	$500,000	$1,413	2.4	$ 40,728,387	57.9%	2.9
	$150	41,350	$ 0.08	$500,000	$1,413	2.2	$ 44,920,706	64.6%	2.7
	$0	20,675	$ 0.08	$500,000	$1,413	5.7	$ 2,997,518	12.7%	11.8
	$15	20,675	$ 0.08	$500,000	$1,413	5.3	$ 5,093,678	14.6%	9.9
	$30	20,675	$ 0.08	$500,000	$1,413	4.9	$ 7,189,837	16.7%	8.6
	$45	20,675	$ 0.08	$500,000	$1,413	4.6	$ 9,285,997	18.7%	7.5
	$60	20,675	$ 0.08	$500,000	$1,413	4.3	$ 11,382,156	20.9%	6.7
	$75	20,675	$ 0.08	$500,000	$1,413	4.1	$ 13,478,315	23.1%	6.0
	$90	20,675	$ 0.08	$500,000	$1,413	3.9	$ 15,574,475	25.3%	5.5
	$105	20,675	$ 0.08	$500,000	$1,413	3.7	$ 17,670,634	27.6%	5.1
	$120	20,675	$ 0.08	$500,000	$1,413	3.5	$ 19,766,793	30.0%	4.7
	$135	20,675	$ 0.08	$500,000	$1,413	3.4	$ 21,862,953	32.5%	4.5
	$150	20,675	$ 0.08	$500,000	$1,413	3.2	$ 23,959,112	35.0%	4.2
	$0	62,652	$ 0.25	$500,000	$1,413	5.7	$ 2,997,518	12.7%	11.8
	$15	62,652	$ 0.25	$500,000	$1,413	4.6	$ 9,349,517	18.8%	7.5
	$30	62,652	$ 0.25	$500,000	$1,413	3.9	$ 15,701,515	25.5%	5.5
	$45	62,652	$ 0.25	$500,000	$1,413	3.3	$ 22,053,513	32.7%	4.4
	$60	62,652	$ 0.25	$500,000	$1,413	2.9	$ 28,405,511	40.5%	3.7
	$75	62,652	$ 0.25	$500,000	$1,413	2.6	$ 34,757,509	49.1%	3.2
	$90	62,652	$ 0.25	$500,000	$1,413	2.4	$ 41,109,507	58.5%	2.8
	$105	62,652	$ 0.25	$500,000	$1,413	2.2	$ 47,461,505	68.9%	2.5
	$120	62,652	$ 0.25	$500,000	$1,413	2.0	$ 53,813,503	80.5%	2.3
	$135	62,652	$ 0.25	$500,000	$1,413	1.8	$ 60,165,501	93.6%	2.1
	$150	62,652	$ 0.25	$500,000	$1,413	1.7	$ 66,517,499	108.5%	1.9
	$0	41,350	$ 0.25	$500,000	$1,413	5.7	$ 2,997,518	12.7%	11.8
	$15	41,350	$ 0.25	$500,000	$1,413	4.9	$ 7,189,837	16.7%	8.6
	$30	41,350	$ 0.25	$500,000	$1,413	4.3	$ 11,382,156	20.9%	6.7
	$45	41,350	$ 0.25	$500,000	$1,413	3.9	$ 15,574,475	25.3%	5.5
	$60	41,350	$ 0.25	$500,000	$1,413	3.5	$ 19,766,793	30.0%	4.7
	$75	41,350	$ 0.25	$500,000	$1,413	3.2	$ 23,959,112	35.0%	4.2
	$90	41,350	$ 0.25	$500,000	$1,413	3.0	$ 28,151,431	40.2%	3.8
	$105	41,350	$ 0.25	$500,000	$1,413	2.7	$ 32,343,750	45.8%	3.4
	$120	41,350	$ 0.25	$500,000	$1,413	2.6	$ 36,536,068	51.6%	3.1
	$135	41,350	$ 0.25	$500,000	$1,413	2.4	$ 40,728,387	57.9%	2.9
	$150	41,350	$ 0.25	$500,000	$1,413	2.2	$ 44,920,706	64.6%	2.7
	$0	20,675	$ 0.25	$500,000	$1,413	5.7	$ 2,997,518	12.7%	11.8
	$15	20,675	$ 0.25	$500,000	$1,413	5.3	$ 5,093,678	14.6%	9.9

	$30	20,675	$	0.25	$500,000	$1,413	4.9	$	7,189,837	16.7%	8.6
	$45	20,675	$	0.25	$500,000	$1,413	4.6	$	9,285,997	18.7%	7.5
	$60	20,675	$	0.25	$500,000	$1,413	4.3	$	11,382,156	20.9%	6.7
	$75	20,675	$	0.25	$500,000	$1,413	4.1	$	13,478,315	23.1%	6.0
	$90	20,675	$	0.25	$500,000	$1,413	3.9	$	15,574,475	25.3%	5.5
	$105	20,675	$	0.25	$500,000	$1,413	3.7	$	17,670,634	27.6%	5.1
	$120	20,675	$	0.25	$500,000	$1,413	3.5	$	19,766,793	30.0%	4.7
	$135	20,675	$	0.25	$500,000	$1,413	3.4	$	21,862,953	32.5%	4.5
	$150	20,675	$	0.25	$500,000	$1,413	3.2	$	23,959,112	35.0%	4.2
Gas Turbine - 45 MW, eGRID (2010) non-baseload baseline -- With Gas Compression	$0	144,784	$	0.08	$500,000	$1,292	6.0	$	2,810,245	11.2%	13.5
	$15	144,784	$	0.08	$500,000	$1,292	4.8	$	17,489,193	17.7%	8.0
	$30	144,784	$	0.08	$500,000	$1,292	4.0	$	32,168,141	24.9%	5.7
	$45	144,784	$	0.08	$500,000	$1,292	3.4	$	46,847,089	32.9%	4.5
	$60	144,784	$	0.08	$500,000	$1,292	3.0	$	61,526,037	41.6%	3.7
	$75	144,784	$	0.08	$500,000	$1,292	2.6	$	76,204,985	51.3%	3.2
	$90	144,784	$	0.08	$500,000	$1,292	2.4	$	90,883,933	62.1%	2.8
	$105	144,784	$	0.08	$500,000	$1,292	2.1	$	105,562,881	74.4%	2.5
	$120	144,784	$	0.08	$500,000	$1,292	2.0	$	120,241,829	88.6%	2.2
	$135	144,784	$	0.08	$500,000	$1,292	1.8	$	134,920,777	105.0%	2.0
	$150	144,784	$	0.08	$500,000	$1,292	1.7	$	149,599,725	124.4%	1.9
	$0	95,557	$	0.08	$500,000	$1,292	6.0	$	2,810,245	11.2%	13.5
	$15	95,557	$	0.08	$500,000	$1,292	5.2	$	12,498,351	15.4%	9.3
	$30	95,557	$	0.08	$500,000	$1,292	4.5	$	22,186,457	19.9%	7.1
	$45	95,557	$	0.08	$500,000	$1,292	4.0	$	31,874,562	24.8%	5.7
	$60	95,557	$	0.08	$500,000	$1,292	3.6	$	41,562,668	29.9%	4.8
	$75	95,557	$	0.08	$500,000	$1,292	3.3	$	51,250,774	35.4%	4.2
	$90	95,557	$	0.08	$500,000	$1,292	3.0	$	60,938,879	41.2%	3.8
	$105	95,557	$	0.08	$500,000	$1,292	2.8	$	70,626,985	47.5%	3.4
	$120	95,557	$	0.08	$500,000	$1,292	2.6	$	80,315,090	54.2%	3.1
	$135	95,557	$	0.08	$500,000	$1,292	2.4	$	90,003,196	61.4%	2.8
	$150	95,557	$	0.08	$500,000	$1,292	2.2	$	99,691,302	69.3%	2.6
	$0	47,779	$	0.08	$500,000	$1,292	6.0	$	2,810,245	11.2%	13.5
	$15	47,779	$	0.08	$500,000	$1,292	5.6	$	7,654,298	13.2%	11.1
	$30	47,779	$	0.08	$500,000	$1,292	5.2	$	12,498,351	15.4%	9.3
	$45	47,779	$	0.08	$500,000	$1,292	4.8	$	17,342,404	17.6%	8.0
	$60	47,779	$	0.08	$500,000	$1,292	4.5	$	22,186,457	19.9%	7.1
	$75	47,779	$	0.08	$500,000	$1,292	4.2	$	27,030,509	22.3%	6.3
	$90	47,779	$	0.08	$500,000	$1,292	4.0	$	31,874,562	24.8%	5.7
	$105	47,779	$	0.08	$500,000	$1,292	3.8	$	36,718,615	27.3%	5.2
	$120	47,779	$	0.08	$500,000	$1,292	3.6	$	41,562,668	29.9%	4.8
	$135	47,779	$	0.08	$500,000	$1,292	3.4	$	46,406,721	32.6%	4.5
	$150	47,779	$	0.08	$500,000	$1,292	3.3	$	51,250,774	35.4%	4.2
	$0	144,784	$	0.25	$500,000	$1,292	6.0	$	2,810,245	11.2%	13.5
	$15	144,784	$	0.25	$500,000	$1,292	4.8	$	17,489,193	17.7%	8.0
	$30	144,784	$	0.25	$500,000	$1,292	4.0	$	32,168,141	24.9%	5.7
	$45	144,784	$	0.25	$500,000	$1,292	3.4	$	46,847,089	32.9%	4.5
	$60	144,784	$	0.25	$500,000	$1,292	3.0	$	61,526,037	41.6%	3.7
	$75	144,784	$	0.25	$500,000	$1,292	2.6	$	76,204,985	51.3%	3.2
	$90	144,784	$	0.25	$500,000	$1,292	2.4	$	90,883,933	62.1%	2.8
	$105	144,784	$	0.25	$500,000	$1,292	2.1	$	105,562,881	74.4%	2.5
	$120	144,784	$	0.25	$500,000	$1,292	2.0	$	120,241,829	88.6%	2.2
	$135	144,784	$	0.25	$500,000	$1,292	1.8	$	134,920,777	105.0%	2.0
	$150	144,784	$	0.25	$500,000	$1,292	1.7	$	149,599,725	124.4%	1.9
	$0	95,557	$	0.25	$500,000	$1,292	6.0	$	2,810,245	11.2%	13.5
	$15	95,557	$	0.25	$500,000	$1,292	5.2	$	12,498,351	15.4%	9.3
	$30	95,557	$	0.25	$500,000	$1,292	4.5	$	22,186,457	19.9%	7.1
	$45	95,557	$	0.25	$500,000	$1,292	4.0	$	31,874,562	24.8%	5.7
	$60	95,557	$	0.25	$500,000	$1,292	3.6	$	41,562,668	29.9%	4.8
	$75	95,557	$	0.25	$500,000	$1,292	3.3	$	51,250,774	35.4%	4.2
	$90	95,557	$	0.25	$500,000	$1,292	3.0	$	60,938,879	41.2%	3.8
	$105	95,557	$	0.25	$500,000	$1,292	2.8	$	70,626,985	47.5%	3.4
	$120	95,557	$	0.25	$500,000	$1,292	2.6	$	80,315,090	54.2%	3.1
	$135	95,557	$	0.25	$500,000	$1,292	2.4	$	90,003,196	61.4%	2.8
	$150	95,557	$	0.25	$500,000	$1,292	2.2	$	99,691,302	69.3%	2.6
	$0	47,779	$	0.25	$500,000	$1,292	6.0	$	2,810,245	11.2%	13.5

	$15	47,779	$ 0.25	$500,000	$1,292	5.6	$ 7,654,298	13.2%	11.1
	$30	47,779	$ 0.25	$500,000	$1,292	5.2	$ 12,498,351	15.4%	9.3
	$45	47,779	$ 0.25	$500,000	$1,292	4.8	$ 17,342,404	17.6%	8.0
	$60	47,779	$ 0.25	$500,000	$1,292	4.5	$ 22,186,457	19.9%	7.1
	$75	47,779	$ 0.25	$500,000	$1,292	4.2	$ 27,030,509	22.3%	6.3
	$90	47,779	$ 0.25	$500,000	$1,292	4.0	$ 31,874,562	24.8%	5.7
	$105	47,779	$ 0.25	$500,000	$1,292	3.8	$ 36,718,615	27.3%	5.2
	$120	47,779	$ 0.25	$500,000	$1,292	3.6	$ 41,562,668	29.9%	4.8
	$135	47,779	$ 0.25	$500,000	$1,292	3.4	$ 46,406,721	32.6%	4.5
	$150	47,779	$ 0.25	$500,000	$1,292	3.3	$ 51,250,774	35.4%	4.2

	2012 eGrid								
Scenario	**NWN CO2e Reduction Incentive ($/tonne/yr)**	**Carbon Emission Reduction**	**ETO Grant Rate ($/kWh)**	**ETO Grant Amount**	**Capital Expenditures ($/kW)**	**Before-Tax Simple Payback**	**Net Present Value**	**Project IRR**	**After-Tax Discounted Payback**
Hospital - 800,000 sf with Two 800 kW Recip Engines, eGRID (2012) non-baseload	$0	4,387	$ 0.08	$317,834	$1,833	8.9	$ (388,404)	4.9%	Exceeds Project Life
	$5	4,387	$ 0.08	$317,834	$1,833	7.7	$ (240,130)	6.8%	Exceeds Project Life
	$10	4,387	$ 0.08	$317,834	$1,833	6.8	$ (91,856)	8.8%	Exceeds Project Life
	$15	4,387	$ 0.08	$317,834	$1,833	6.1	$ 56,418	10.8%	13.3
	$20	4,387	$ 0.08	$317,834	$1,833	5.5	$ 204,692	12.7%	9.7
	$25	4,387	$ 0.08	$317,834	$1,833	5.1	$ 352,966	14.7%	8.2
	$30	4,387	$ 0.08	$317,834	$1,833	4.7	$ 501,241	16.7%	7.1
	$35	4,387	$ 0.08	$317,834	$1,833	4.3	$ 649,515	18.7%	6.2
	$40	4,387	$ 0.08	$317,834	$1,833	4.0	$ 797,789	20.7%	5.6
	$45	4,387	$ 0.08	$317,834	$1,833	3.8	$ 946,063	22.8%	5.1
	$50	4,387	$ 0.08	$317,834	$1,833	3.5	$ 1,094,337	24.8%	4.8
	$0	2,896	$ 0.08	$317,834	$1,833	8.9	$ (388,404)	4.9%	Exceeds Project Life
	$5	2,896	$ 0.08	$317,834	$1,833	8.1	$ (290,544)	6.1%	Exceeds Project Life
	$10	2,896	$ 0.08	$317,834	$1,833	7.4	$ (192,683)	7.4%	Exceeds Project Life
	$15	2,896	$ 0.08	$317,834	$1,833	6.8	$ (94,822)	8.7%	Exceeds Project Life
	$20	2,896	$ 0.08	$317,834	$1,833	6.4	$ 3,039	10.0%	14.9
	$25	2,896	$ 0.08	$317,834	$1,833	5.9	$ 100,900	11.3%	12.0
	$30	2,896	$ 0.08	$317,834	$1,833	5.6	$ 198,761	12.7%	9.8
	$35	2,896	$ 0.08	$317,834	$1,833	5.2	$ 296,622	14.0%	8.7
	$40	2,896	$ 0.08	$317,834	$1,833	4.9	$ 394,483	15.3%	7.8
	$45	2,896	$ 0.08	$317,834	$1,833	4.7	$ 492,344	16.6%	7.1
	$50	2,896	$ 0.08	$317,834	$1,833	4.5	$ 590,205	17.9%	6.5
	$0	1,448	$ 0.08	$317,834	$1,833	8.9	$ (388,404)	4.9%	Exceeds Project Life
	$5	1,448	$ 0.08	$317,834	$1,833	8.5	$ (339,474)	5.5%	Exceeds Project Life
	$10	1,448	$ 0.08	$317,834	$1,833	8.1	$ (290,544)	6.1%	Exceeds Project Life
	$15	1,448	$ 0.08	$317,834	$1,833	7.7	$ (241,613)	6.8%	Exceeds Project Life
	$20	1,448	$ 0.08	$317,834	$1,833	7.4	$ (192,683)	7.4%	Exceeds Project Life
	$25	1,448	$ 0.08	$317,834	$1,833	7.1	$ (143,752)	8.1%	Exceeds Project Life
	$30	1,448	$ 0.08	$317,834	$1,833	6.8	$ (94,822)	8.7%	Exceeds Project Life
	$35	1,448	$ 0.08	$317,834	$1,833	6.6	$ (45,891)	9.4%	Exceeds Project Life
	$40	1,448	$ 0.08	$317,834	$1,833	6.4	$ 3,039	10.0%	14.9
	$45	1,448	$ 0.08	$317,834	$1,833	6.1	$ 51,970	10.7%	13.4
	$50	1,448	$ 0.08	$317,834	$1,833	5.9	$ 100,900	11.3%	12.0
	$0	4,387	$ 0.25	$500,000	$1,833	7.6	$ (229,608)	6.8%	Exceeds Project Life
	$5	4,387	$ 0.25	$317,834	$1,833	7.7	$ (240,130)	6.8%	Exceeds Project Life
	$10	4,387	$ 0.25	$317,834	$1,833	6.8	$ (91,856)	8.8%	Exceeds Project Life
	$15	4,387	$ 0.25	$317,834	$1,833	6.1	$ 56,418	10.8%	13.3
	$20	4,387	$ 0.25	$317,834	$1,833	5.5	$ 204,692	12.7%	9.7
	$25	4,387	$ 0.25	$317,834	$1,833	5.1	$ 352,966	14.7%	8.2
	$30	4,387	$ 0.25	$317,834	$1,833	4.7	$ 501,241	16.7%	7.1
	$35	4,387	$ 0.25	$317,834	$1,833	4.3	$ 649,515	18.7%	6.2
	$40	4,387	$ 0.25	$317,834	$1,833	4.0	$ 797,789	20.7%	5.6
	$45	4,387	$ 0.25	$317,834	$1,833	3.8	$ 946,063	22.8%	5.1
	$50	4,387	$ 0.25	$317,834	$1,833	3.5	$ 1,094,337	24.8%	4.8
	$0	2,896	$ 0.25	$317,834	$1,833	8.9	$ (388,404)	4.9%	Exceeds Project Life
	$5	2,896	$ 0.25	$317,834	$1,833	8.1	$ (290,544)	6.1%	Exceeds Project Life

	$10	2,896	$	0.25	$317,834	$1,833	7.4	$	(192,683)	7.4%	Exceeds Project Life
	$15	2,896	$	0.25	$317,834	$1,833	6.8	$	(94,822)	8.7%	Exceeds Project Life
	$20	2,896	$	0.25	$317,834	$1,833	6.4	$	3,039	10.0%	14.9
	$25	2,896	$	0.25	$317,834	$1,833	5.9	$	100,900	11.3%	12.0
	$30	2,896	$	0.25	$317,834	$1,833	5.6	$	198,761	12.7%	9.8
	$35	2,896	$	0.25	$317,834	$1,833	5.2	$	296,622	14.0%	8.7
	$40	2,896	$	0.25	$317,834	$1,833	4.9	$	394,483	15.3%	7.8
	$45	2,896	$	0.25	$317,834	$1,833	4.7	$	492,344	16.6%	7.1
	$50	2,896	$	0.25	$317,834	$1,833	4.5	$	590,205	17.9%	6.5
	$0	1,448	$	0.25	$317,834	$1,833	8.9	$	(388,404)	4.9%	Exceeds Project Life
	$5	1,448	$	0.25	$317,834	$1,833	8.5	$	(339,474)	5.5%	Exceeds Project Life
	$10	1,448	$	0.25	$317,834	$1,833	8.1	$	(290,544)	6.1%	Exceeds Project Life
	$15	1,448	$	0.25	$317,834	$1,833	7.7	$	(241,613)	6.8%	Exceeds Project Life
	$20	1,448	$	0.25	$317,834	$1,833	7.4	$	(192,683)	7.4%	Exceeds Project Life
	$25	1,448	$	0.25	$317,834	$1,833	7.1	$	(143,752)	8.1%	Exceeds Project Life
	$30	1,448	$	0.25	$317,834	$1,833	6.8	$	(94,822)	8.7%	Exceeds Project Life
	$35	1,448	$	0.25	$317,834	$1,833	6.6	$	(45,891)	9.4%	Exceeds Project Life
	$40	1,448	$	0.25	$317,834	$1,833	6.4	$	3,039	10.0%	14.9
	$45	1,448	$	0.25	$317,834	$1,833	6.1	$	51,970	10.7%	13.4
	$50	1,448	$	0.25	$317,834	$1,833	5.9	$	100,900	11.3%	12.0
Reciprocating Engine - 500 kW, eGRID (2012) non-baseload baseline	$0	1,782	$	0.08	$110,183	$1,925	8.7	$	(119,706)	5.2%	Exceeds Project Life
	$5	1,782	$	0.08	$110,183	$1,925	7.4	$	(59,472)	7.6%	Exceeds Project Life
	$10	1,782	$	0.08	$110,183	$1,925	6.4	$	761	10.0%	14.9
	$15	1,782	$	0.08	$110,183	$1,925	5.6	$	60,995	12.5%	10.0
	$20	1,782	$	0.08	$110,183	$1,925	5.0	$	121,228	15.0%	8.0
	$25	1,782	$	0.08	$110,183	$1,925	4.5	$	181,462	17.4%	6.7
	$30	1,782	$	0.08	$110,183	$1,925	4.1	$	241,695	19.9%	5.8
	$35	1,782	$	0.08	$110,183	$1,925	3.8	$	301,929	22.4%	5.2
	$40	1,782	$	0.08	$110,183	$1,925	3.5	$	362,162	25.0%	4.8
	$45	1,782	$	0.08	$110,183	$1,925	3.3	$	422,396	27.5%	4.5
	$50	1,782	$	0.08	$110,183	$1,925	3.1	$	482,629	30.1%	4.2
	$0	1,176	$	0.08	$110,183	$1,925	8.7	$	(119,706)	5.2%	Exceeds Project Life
	$5	1,176	$	0.08	$110,183	$1,925	7.8	$	(79,952)	6.8%	Exceeds Project Life
	$10	1,176	$	0.08	$110,183	$1,925	7.0	$	(40,198)	8.4%	Exceeds Project Life
	$15	1,176	$	0.08	$110,183	$1,925	6.4	$	(444)	10.0%	Exceeds Project Life
	$20	1,176	$	0.08	$110,183	$1,925	5.9	$	39,311	11.6%	11.6
	$25	1,176	$	0.08	$110,183	$1,925	5.4	$	79,065	13.2%	9.3
	$30	1,176	$	0.08	$110,183	$1,925	5.0	$	118,819	14.9%	8.1
	$35	1,176	$	0.08	$110,183	$1,925	4.7	$	158,573	16.5%	7.2
	$40	1,176	$	0.08	$110,183	$1,925	4.4	$	198,327	18.1%	6.4
	$45	1,176	$	0.08	$110,183	$1,925	4.2	$	238,081	19.8%	5.9
	$50	1,176	$	0.08	$110,183	$1,925	3.9	$	277,835	21.4%	5.4
	$0	588	$	0.08	$110,183	$1,925	8.7	$	(119,706)	5.2%	Exceeds Project Life
	$5	588	$	0.08	$110,183	$1,925	8.2	$	(99,829)	6.0%	Exceeds Project Life
	$10	588	$	0.08	$110,183	$1,925	7.8	$	(79,952)	6.8%	Exceeds Project Life
	$15	588	$	0.08	$110,183	$1,925	7.4	$	(60,075)	7.6%	Exceeds Project Life
	$20	588	$	0.08	$110,183	$1,925	7.0	$	(40,198)	8.4%	Exceeds Project Life
	$25	588	$	0.08	$110,183	$1,925	6.7	$	(20,321)	9.2%	Exceeds Project Life
	$30	588	$	0.08	$110,183	$1,925	6.4	$	(444)	10.0%	Exceeds Project Life
	$35	588	$	0.08	$110,183	$1,925	6.1	$	19,434	10.8%	13.2
	$40	588	$	0.08	$110,183	$1,925	5.9	$	39,311	11.6%	11.6
	$45	588	$	0.08	$110,183	$1,925	5.6	$	59,188	12.4%	10.1
	$50	588	$	0.08	$110,183	$1,925	5.4	$	79,065	13.2%	9.3
	$0	1,782	$	0.25	$344,323	$1,925	3.9	$	84,397	14.2%	8.4
	$5	1,782	$	0.25	$110,183	$1,925	7.4	$	(59,472)	7.6%	Exceeds Project Life
	$10	1,782	$	0.25	$110,183	$1,925	6.4	$	761	10.0%	14.9
	$15	1,782	$	0.25	$110,183	$1,925	5.6	$	60,995	12.5%	10.0
	$20	1,782	$	0.25	$110,183	$1,925	5.0	$	121,228	15.0%	8.0
	$25	1,782	$	0.25	$110,183	$1,925	4.5	$	181,462	17.4%	6.7
	$30	1,782	$	0.25	$110,183	$1,925	4.1	$	241,695	19.9%	5.8
	$35	1,782	$	0.25	$110,183	$1,925	3.8	$	301,929	22.4%	5.2
	$40	1,782	$	0.25	$110,183	$1,925	3.5	$	362,162	25.0%	4.8
	$45	1,782	$	0.25	$110,183	$1,925	3.3	$	422,396	27.5%	4.5
	$50	1,782	$	0.25	$110,183	$1,925	3.1	$	482,629	30.1%	4.2
	$0	1,176	$	0.25	$110,183	$1,925	8.7	$	(119,706)	5.2%	Exceeds Project Life

	$5	1,176	$	0.25	$110,183	$1,925	7.8	$	(79,952)	6.8%	Exceeds Project Life
	$10	1,176	$	0.25	$110,183	$1,925	7.0	$	(40,198)	8.4%	Exceeds Project Life
	$15	1,176	$	0.25	$110,183	$1,925	6.4	$	(444)	10.0%	Exceeds Project Life
	$20	1,176	$	0.25	$110,183	$1,925	5.9	$	39,311	11.6%	11.6
	$25	1,176	$	0.25	$110,183	$1,925	5.4	$	79,065	13.2%	9.3
	$30	1,176	$	0.25	$110,183	$1,925	5.0	$	118,819	14.9%	8.1
	$35	1,176	$	0.25	$110,183	$1,925	4.7	$	158,573	16.5%	7.2
	$40	1,176	$	0.25	$110,183	$1,925	4.4	$	198,327	18.1%	6.4
	$45	1,176	$	0.25	$110,183	$1,925	4.2	$	238,081	19.8%	5.9
	$50	1,176	$	0.25	$110,183	$1,925	3.9	$	277,835	21.4%	5.4
	$0	588	$	0.25	$110,183	$1,925	8.7	$	(119,706)	5.2%	Exceeds Project Life
	$5	588	$	0.25	$110,183	$1,925	8.2	$	(99,829)	6.0%	Exceeds Project Life
	$10	588	$	0.25	$110,183	$1,925	7.8	$	(79,952)	6.8%	Exceeds Project Life
	$15	588	$	0.25	$110,183	$1,925	7.4	$	(60,075)	7.6%	Exceeds Project Life
	$20	588	$	0.25	$110,183	$1,925	7.0	$	(40,198)	8.4%	Exceeds Project Life
	$25	588	$	0.25	$110,183	$1,925	6.7	$	(20,321)	9.2%	Exceeds Project Life
	$30	588	$	0.25	$110,183	$1,925	6.4	$	(444)	10.0%	Exceeds Project Life
	$35	588	$	0.25	$110,183	$1,925	6.1	$	19,434	10.8%	13.2
	$40	588	$	0.25	$110,183	$1,925	5.9	$	39,311	11.6%	11.6
	$45	588	$	0.25	$110,183	$1,925	5.6	$	59,188	12.4%	10.1
	$50	588	$	0.25	$110,183	$1,925	5.4	$	79,065	13.2%	9.3
Reciprocating Engine - 4.3 MW, eGRID (2012) non-baseload baseline	$0	19,224	$	0.08	$500,000	$1,656	3.9	$	1,881,269	18.7%	7.1
	$5	19,224	$	0.08	$500,000	$1,656	3.5	$	2,530,953	21.9%	5.7
	$10	19,224	$	0.08	$500,000	$1,656	3.2	$	3,180,637	25.2%	4.9
	$15	19,224	$	0.08	$500,000	$1,656	2.9	$	3,830,321	28.6%	4.5
	$20	19,224	$	0.08	$500,000	$1,656	2.7	$	4,480,005	32.1%	4.2
	$25	19,224	$	0.08	$500,000	$1,656	2.5	$	5,129,689	35.7%	3.9
	$30	19,224	$	0.08	$500,000	$1,656	2.4	$	5,779,373	39.5%	3.6
	$35	19,224	$	0.08	$500,000	$1,656	2.2	$	6,429,057	43.3%	3.4
	$40	19,224	$	0.08	$500,000	$1,656	2.1	$	7,078,741	47.3%	3.2
	$45	19,224	$	0.08	$500,000	$1,656	2.0	$	7,728,425	51.4%	3.0
	$50	19,224	$	0.08	$500,000	$1,656	1.9	$	8,378,109	55.7%	2.9
	$0	12,688	$	0.08	$500,000	$1,656	3.9	$	1,881,269	18.7%	7.1
	$5	12,688	$	0.08	$500,000	$1,656	3.7	$	2,310,061	20.8%	6.1
	$10	12,688	$	0.08	$500,000	$1,656	3.4	$	2,738,852	22.9%	5.4
	$15	12,688	$	0.08	$500,000	$1,656	3.2	$	3,167,643	25.1%	4.9
	$20	12,688	$	0.08	$500,000	$1,656	3.0	$	3,596,435	27.3%	4.7
	$25	12,688	$	0.08	$500,000	$1,656	2.9	$	4,025,226	29.6%	4.4
	$30	12,688	$	0.08	$500,000	$1,656	2.7	$	4,454,018	32.0%	4.2
	$35	12,688	$	0.08	$500,000	$1,656	2.6	$	4,882,809	34.3%	4.0
	$40	12,688	$	0.08	$500,000	$1,656	2.5	$	5,311,600	36.8%	3.8
	$45	12,688	$	0.08	$500,000	$1,656	2.4	$	5,740,392	39.2%	3.6
	$50	12,688	$	0.08	$500,000	$1,656	2.3	$	6,169,183	41.8%	3.5
	$0	6,344	$	0.08	$500,000	$1,656	3.9	$	1,881,269	18.7%	7.1
	$5	6,344	$	0.08	$500,000	$1,656	3.8	$	2,095,665	19.7%	6.6
	$10	6,344	$	0.08	$500,000	$1,656	3.7	$	2,310,061	20.8%	6.1
	$15	6,344	$	0.08	$500,000	$1,656	3.5	$	2,524,456	21.8%	5.8
	$20	6,344	$	0.08	$500,000	$1,656	3.4	$	2,738,852	22.9%	5.4
	$25	6,344	$	0.08	$500,000	$1,656	3.3	$	2,953,248	24.0%	5.2
	$30	6,344	$	0.08	$500,000	$1,656	3.2	$	3,167,643	25.1%	4.9
	$35	6,344	$	0.08	$500,000	$1,656	3.1	$	3,382,039	26.2%	4.8
	$40	6,344	$	0.08	$500,000	$1,656	3.0	$	3,596,435	27.3%	4.7
	$45	6,344	$	0.08	$500,000	$1,656	3.0	$	3,810,831	28.5%	4.5
	$50	6,344	$	0.08	$500,000	$1,656	2.9	$	4,025,226	29.6%	4.4
	$0	19,224	$	0.25	$500,000	$1,656	3.9	$	1,881,269	18.7%	7.1
	$5	19,224	$	0.25	$500,000	$1,656	3.5	$	2,530,953	21.9%	5.7
	$10	19,224	$	0.25	$500,000	$1,656	3.2	$	3,180,637	25.2%	4.9
	$15	19,224	$	0.25	$500,000	$1,656	2.9	$	3,830,321	28.6%	4.5
	$20	19,224	$	0.25	$500,000	$1,656	2.7	$	4,480,005	32.1%	4.2
	$25	19,224	$	0.25	$500,000	$1,656	2.5	$	5,129,689	35.7%	3.9
	$30	19,224	$	0.25	$500,000	$1,656	2.4	$	5,779,373	39.5%	3.6
	$35	19,224	$	0.25	$500,000	$1,656	2.2	$	6,429,057	43.3%	3.4
	$40	19,224	$	0.25	$500,000	$1,656	2.1	$	7,078,741	47.3%	3.2
	$45	19,224	$	0.25	$500,000	$1,656	2.0	$	7,728,425	51.4%	3.0
	$50	19,224	$	0.25	$500,000	$1,656	1.9	$	8,378,109	55.7%	2.9

	$0	12,688	$ 0.25	$500,000	$1,656	3.9	$ 1,881,269	18.7%	7.1
	$5	12,688	$ 0.25	$500,000	$1,656	3.7	$ 2,310,061	20.8%	6.1
	$10	12,688	$ 0.25	$500,000	$1,656	3.4	$ 2,738,852	22.9%	5.4
	$15	12,688	$ 0.25	$500,000	$1,656	3.2	$ 3,167,643	25.1%	4.9
	$20	12,688	$ 0.25	$500,000	$1,656	3.0	$ 3,596,435	27.3%	4.7
	$25	12,688	$ 0.25	$500,000	$1,656	2.9	$ 4,025,226	29.6%	4.4
	$30	12,688	$ 0.25	$500,000	$1,656	2.7	$ 4,454,018	32.0%	4.2
	$35	12,688	$ 0.25	$500,000	$1,656	2.6	$ 4,882,809	34.3%	4.0
	$40	12,688	$ 0.25	$500,000	$1,656	2.5	$ 5,311,600	36.8%	3.8
	$45	12,688	$ 0.25	$500,000	$1,656	2.4	$ 5,740,392	39.2%	3.6
	$50	12,688	$ 0.25	$500,000	$1,656	2.3	$ 6,169,183	41.8%	3.5
	$0	6,344	$ 0.25	$500,000	$1,656	3.9	$ 1,881,269	18.7%	7.1
	$5	6,344	$ 0.25	$500,000	$1,656	3.8	$ 2,095,665	19.7%	6.6
	$10	6,344	$ 0.25	$500,000	$1,656	3.7	$ 2,310,061	20.8%	6.1
	$15	6,344	$ 0.25	$500,000	$1,656	3.5	$ 2,524,456	21.8%	5.8
	$20	6,344	$ 0.25	$500,000	$1,656	3.4	$ 2,738,852	22.9%	5.4
	$25	6,344	$ 0.25	$500,000	$1,656	3.3	$ 2,953,248	24.0%	5.2
	$30	6,344	$ 0.25	$500,000	$1,656	3.2	$ 3,167,643	25.1%	4.9
	$35	6,344	$ 0.25	$500,000	$1,656	3.1	$ 3,382,039	26.2%	4.8
	$40	6,344	$ 0.25	$500,000	$1,656	3.0	$ 3,596,435	27.3%	4.7
	$45	6,344	$ 0.25	$500,000	$1,656	3.0	$ 3,810,831	28.5%	4.5
	$50	6,344	$ 0.25	$500,000	$1,656	2.9	$ 4,025,226	29.6%	4.4
	$0	68,399	$ 0.08	$500,000	$1,413	5.7	$ 2,997,518	12.7%	11.8
	$5	68,399	$ 0.08	$500,000	$1,413	5.2	$ 5,309,069	14.8%	9.8
	$10	68,399	$ 0.08	$500,000	$1,413	4.9	$ 7,620,619	17.1%	8.3
	$15	68,399	$ 0.08	$500,000	$1,413	4.5	$ 9,932,169	19.4%	7.2
	$20	68,399	$ 0.08	$500,000	$1,413	4.2	$ 12,243,719	21.8%	6.4
	$25	68,399	$ 0.08	$500,000	$1,413	4.0	$ 14,555,269	24.2%	5.7
	$30	68,399	$ 0.08	$500,000	$1,413	3.8	$ 16,866,820	26.8%	5.2
	$35	68,399	$ 0.08	$500,000	$1,413	3.6	$ 19,178,370	29.4%	4.8
	$40	68,399	$ 0.08	$500,000	$1,413	3.4	$ 21,489,920	32.0%	4.5
	$45	68,399	$ 0.08	$500,000	$1,413	3.2	$ 23,801,470	34.8%	4.2
	$50	68,399	$ 0.08	$500,000	$1,413	3.1	$ 26,113,020	37.6%	3.9
	$0	45,143	$ 0.08	$500,000	$1,413	5.7	$ 2,997,518	12.7%	11.8
	$5	45,143	$ 0.08	$500,000	$1,413	5.4	$ 4,523,142	14.1%	10.4
	$10	45,143	$ 0.08	$500,000	$1,413	5.1	$ 6,048,765	15.6%	9.3
	$15	45,143	$ 0.08	$500,000	$1,413	4.9	$ 7,574,388	17.0%	8.3
	$20	45,143	$ 0.08	$500,000	$1,413	4.6	$ 9,100,011	18.5%	7.6
	$25	45,143	$ 0.08	$500,000	$1,413	4.4	$ 10,625,634	20.1%	6.9
	$30	45,143	$ 0.08	$500,000	$1,413	4.3	$ 12,151,257	21.7%	6.4
	$35	45,143	$ 0.08	$500,000	$1,413	4.1	$ 13,676,880	23.3%	6.0
	$40	45,143	$ 0.08	$500,000	$1,413	3.9	$ 15,202,503	24.9%	5.6
	$45	45,143	$ 0.08	$500,000	$1,413	3.8	$ 16,728,127	26.6%	5.3
	$50	45,143	$ 0.08	$500,000	$1,413	3.6	$ 18,253,750	28.3%	5.0
	$0	22,572	$ 0.08	$500,000	$1,413	5.7	$ 2,997,518	12.7%	11.8
	$5	22,572	$ 0.08	$500,000	$1,413	5.5	$ 3,760,330	13.4%	11.1
	$10	22,572	$ 0.08	$500,000	$1,413	5.4	$ 4,523,142	14.1%	10.4
	$15	22,572	$ 0.08	$500,000	$1,413	5.2	$ 5,285,953	14.8%	9.8
	$20	22,572	$ 0.08	$500,000	$1,413	5.1	$ 6,048,765	15.6%	9.3
	$25	22,572	$ 0.08	$500,000	$1,413	5.0	$ 6,811,576	16.3%	8.8
	$30	22,572	$ 0.08	$500,000	$1,413	4.9	$ 7,574,388	17.0%	8.3
	$35	22,572	$ 0.08	$500,000	$1,413	4.7	$ 8,337,199	17.8%	7.9
	$40	22,572	$ 0.08	$500,000	$1,413	4.6	$ 9,100,011	18.5%	7.6
	$45	22,572	$ 0.08	$500,000	$1,413	4.5	$ 9,862,823	19.3%	7.3
Gas Turbine - 21.7 MW, eGRID (2012) non-baseload baseline	$50	22,572	$ 0.08	$500,000	$1,413	4.4	$ 10,625,634	20.1%	6.9
	$0	68,399	$ 0.25	$500,000	$1,413	5.7	$ 2,997,518	12.7%	11.8
	$5	68,399	$ 0.25	$500,000	$1,413	5.2	$ 5,309,069	14.8%	9.8
	$10	68,399	$ 0.25	$500,000	$1,413	4.9	$ 7,620,619	17.1%	8.3
	$15	68,399	$ 0.25	$500,000	$1,413	4.5	$ 9,932,169	19.4%	7.2
	$20	68,399	$ 0.25	$500,000	$1,413	4.2	$ 12,243,719	21.8%	6.4
	$25	68,399	$ 0.25	$500,000	$1,413	4.0	$ 14,555,269	24.2%	5.7
	$30	68,399	$ 0.25	$500,000	$1,413	3.8	$ 16,866,820	26.8%	5.2
	$35	68,399	$ 0.25	$500,000	$1,413	3.6	$ 19,178,370	29.4%	4.8
	$40	68,399	$ 0.25	$500,000	$1,413	3.4	$ 21,489,920	32.0%	4.5
	$45	68,399	$ 0.25	$500,000	$1,413	3.2	$ 23,801,470	34.8%	4.2

	$50	68,399	$	0.25	$500,000	$1,413	3.1	$	26,113,020	37.6%	3.9
	$0	45,143	$	0.25	$500,000	$1,413	5.7	$	2,997,518	12.7%	11.8
	$5	45,143	$	0.25	$500,000	$1,413	5.4	$	4,523,142	14.1%	10.4
	$10	45,143	$	0.25	$500,000	$1,413	5.1	$	6,048,765	15.6%	9.3
	$15	45,143	$	0.25	$500,000	$1,413	4.9	$	7,574,388	17.0%	8.3
	$20	45,143	$	0.25	$500,000	$1,413	4.6	$	9,100,011	18.5%	7.6
	$25	45,143	$	0.25	$500,000	$1,413	4.4	$	10,625,634	20.1%	6.9
	$30	45,143	$	0.25	$500,000	$1,413	4.3	$	12,151,257	21.7%	6.4
	$35	45,143	$	0.25	$500,000	$1,413	4.1	$	13,676,880	23.3%	6.0
	$40	45,143	$	0.25	$500,000	$1,413	3.9	$	15,202,503	24.9%	5.6
	$45	45,143	$	0.25	$500,000	$1,413	3.8	$	16,728,127	26.6%	5.3
	$50	45,143	$	0.25	$500,000	$1,413	3.6	$	18,253,750	28.3%	5.0
	$0	22,572	$	0.25	$500,000	$1,413	5.7	$	2,997,518	12.7%	11.8
	$5	22,572	$	0.25	$500,000	$1,413	5.5	$	3,760,330	13.4%	11.1
	$10	22,572	$	0.25	$500,000	$1,413	5.4	$	4,523,142	14.1%	10.4
	$15	22,572	$	0.25	$500,000	$1,413	5.2	$	5,285,953	14.8%	9.8
	$20	22,572	$	0.25	$500,000	$1,413	5.1	$	6,048,765	15.6%	9.3
	$25	22,572	$	0.25	$500,000	$1,413	5.0	$	6,811,576	16.3%	8.8
	$30	22,572	$	0.25	$500,000	$1,413	4.9	$	7,574,388	17.0%	8.3
	$35	22,572	$	0.25	$500,000	$1,413	4.7	$	8,337,199	17.8%	7.9
	$40	22,572	$	0.25	$500,000	$1,413	4.6	$	9,100,011	18.5%	7.6
	$45	22,572	$	0.25	$500,000	$1,413	4.5	$	9,862,823	19.3%	7.3
	$50	22,572	$	0.25	$500,000	$1,413	4.4	$	10,625,634	20.1%	6.9
Gas Turbine - 45 MW, eGRID (2012) non-baseload baseline	$0	144,784	$	0.08	$500,000	$1,292	6.0	$	2,810,245	11.2%	13.5
	$5	144,784	$	0.08	$500,000	$1,292	5.6	$	7,703,228	13.3%	11.1
	$10	144,784	$	0.08	$500,000	$1,292	5.1	$	12,596,211	15.4%	9.3
	$15	144,784	$	0.08	$500,000	$1,292	4.8	$	17,489,193	17.7%	8.0
	$20	144,784	$	0.08	$500,000	$1,292	4.5	$	22,382,176	20.0%	7.0
	$25	144,784	$	0.08	$500,000	$1,292	4.2	$	27,275,159	22.4%	6.3
	$30	144,784	$	0.08	$500,000	$1,292	4.0	$	32,168,141	24.9%	5.7
	$35	144,784	$	0.08	$500,000	$1,292	3.8	$	37,061,124	27.5%	5.2
	$40	144,784	$	0.08	$500,000	$1,292	3.6	$	41,954,106	30.1%	4.8
	$45	144,784	$	0.08	$500,000	$1,292	3.4	$	46,847,089	32.9%	4.5
	$50	144,784	$	0.08	$500,000	$1,292	3.2	$	51,740,072	35.7%	4.2
	$0	95,557	$	0.08	$500,000	$1,292	6.0	$	2,810,245	11.2%	13.5
	$5	95,557	$	0.08	$500,000	$1,292	5.7	$	6,039,614	12.5%	11.9
	$10	95,557	$	0.08	$500,000	$1,292	5.4	$	9,268,982	14.0%	10.5
	$15	95,557	$	0.08	$500,000	$1,292	5.2	$	12,498,351	15.4%	9.3
	$20	95,557	$	0.08	$500,000	$1,292	4.9	$	15,727,720	16.9%	8.4
	$25	95,557	$	0.08	$500,000	$1,292	4.7	$	18,957,088	18.4%	7.7
	$30	95,557	$	0.08	$500,000	$1,292	4.5	$	22,186,457	19.9%	7.1
	$35	95,557	$	0.08	$500,000	$1,292	4.3	$	25,415,825	21.5%	6.5
	$40	95,557	$	0.08	$500,000	$1,292	4.1	$	28,645,194	23.1%	6.1
	$45	95,557	$	0.08	$500,000	$1,292	4.0	$	31,874,562	24.8%	5.7
	$50	95,557	$	0.08	$500,000	$1,292	3.8	$	35,103,931	26.4%	5.4
	$0	47,779	$	0.08	$500,000	$1,292	6.0	$	2,810,245	11.2%	13.5
	$5	47,779	$	0.08	$500,000	$1,292	5.9	$	4,424,930	11.9%	12.7
	$10	47,779	$	0.08	$500,000	$1,292	5.7	$	6,039,614	12.5%	11.9
	$15	47,779	$	0.08	$500,000	$1,292	5.6	$	7,654,298	13.2%	11.1
	$20	47,779	$	0.08	$500,000	$1,292	5.4	$	9,268,982	14.0%	10.5
	$25	47,779	$	0.08	$500,000	$1,292	5.3	$	10,883,667	14.7%	9.8
	$30	47,779	$	0.08	$500,000	$1,292	5.2	$	12,498,351	15.4%	9.3
	$35	47,779	$	0.08	$500,000	$1,292	5.0	$	14,113,035	16.1%	8.8
	$40	47,779	$	0.08	$500,000	$1,292	4.9	$	15,727,720	16.9%	8.4
	$45	47,779	$	0.08	$500,000	$1,292	4.8	$	17,342,404	17.6%	8.0
	$50	47,779	$	0.08	$500,000	$1,292	4.7	$	18,957,088	18.4%	7.7
	$0	144,784	$	0.25	$500,000	$1,292	6.0	$	2,810,245	11.2%	13.5
	$5	144,784	$	0.25	$500,000	$1,292	5.6	$	7,703,228	13.3%	11.1
	$10	144,784	$	0.25	$500,000	$1,292	5.1	$	12,596,211	15.4%	9.3
	$15	144,784	$	0.25	$500,000	$1,292	4.8	$	17,489,193	17.7%	8.0
	$20	144,784	$	0.25	$500,000	$1,292	4.5	$	22,382,176	20.0%	7.0
	$25	144,784	$	0.25	$500,000	$1,292	4.2	$	27,275,159	22.4%	6.3
	$30	144,784	$	0.25	$500,000	$1,292	4.0	$	32,168,141	24.9%	5.7
	$35	144,784	$	0.25	$500,000	$1,292	3.8	$	37,061,124	27.5%	5.2
	$40	144,784	$	0.25	$500,000	$1,292	3.6	$	41,954,106	30.1%	4.8

	$45	144,784	$	0.25	$500,000	$1,292	3.4	$	46,847,089	32.9%	4.5
	$50	144,784	$	0.25	$500,000	$1,292	3.2	$	51,740,072	35.7%	4.2
	$0	95,557	$	0.25	$500,000	$1,292	6.0	$	2,810,245	11.2%	13.5
	$5	95,557	$	0.25	$500,000	$1,292	5.7	$	6,039,614	12.5%	11.9
	$10	95,557	$	0.25	$500,000	$1,292	5.4	$	9,268,982	14.0%	10.5
	$15	95,557	$	0.25	$500,000	$1,292	5.2	$	12,498,351	15.4%	9.3
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	$25	95,557	$	0.25	$500,000	$1,292	4.7	$	18,957,088	18.4%	7.7
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	$50	95,557	$	0.25	$500,000	$1,292	3.8	$	35,103,931	26.4%	5.4
	$0	47,779	$	0.25	$500,000	$1,292	6.0	$	2,810,245	11.2%	13.5
	$5	47,779	$	0.25	$500,000	$1,292	5.9	$	4,424,930	11.9%	12.7
	$10	47,779	$	0.25	$500,000	$1,292	5.7	$	6,039,614	12.5%	11.9
	$15	47,779	$	0.25	$500,000	$1,292	5.6	$	7,654,298	13.2%	11.1
	$20	47,779	$	0.25	$500,000	$1,292	5.4	$	9,268,982	14.0%	10.5
	$25	47,779	$	0.25	$500,000	$1,292	5.3	$	10,883,667	14.7%	9.8
	$30	47,779	$	0.25	$500,000	$1,292	5.2	$	12,498,351	15.4%	9.3
	$35	47,779	$	0.25	$500,000	$1,292	5.0	$	14,113,035	16.1%	8.8
	$40	47,779	$	0.25	$500,000	$1,292	4.9	$	15,727,720	16.9%	8.4
	$45	47,779	$	0.25	$500,000	$1,292	4.8	$	17,342,404	17.6%	8.0
	$50	47,779	$	0.25	$500,000	$1,292	4.7	$	18,957,088	18.4%	7.7
Gas Turbine - 45 MW, eGRID (2012) non-baseload baseline, 70% CapEx	$0	144,784	$	0.08	$500,000	$905	4.0	$	17,735,773	20.6%	7.3
	$5	144,784	$	0.08	$500,000	$905	3.6	$	22,628,756	24.0%	6.1
	$10	144,784	$	0.08	$500,000	$905	3.4	$	27,521,739	27.6%	5.3
	$15	144,784	$	0.08	$500,000	$905	3.1	$	32,414,721	31.3%	4.7
	$20	144,784	$	0.08	$500,000	$905	2.9	$	37,307,704	35.3%	4.3
	$25	144,784	$	0.08	$500,000	$905	2.8	$	42,200,687	39.5%	3.9
	$30	144,784	$	0.08	$500,000	$905	2.6	$	47,093,669	44.0%	3.6
	$35	144,784	$	0.08	$500,000	$905	2.5	$	51,986,652	48.7%	3.3
	$40	144,784	$	0.08	$500,000	$905	2.3	$	56,879,635	53.6%	3.1
	$45	144,784	$	0.08	$500,000	$905	2.2	$	61,772,617	58.9%	2.9
	$50	144,784	$	0.08	$500,000	$905	2.1	$	66,665,600	64.5%	2.7
	$0	95,557	$	0.08	$500,000	$905	4.0	$	17,735,773	20.6%	7.3
	$5	95,557	$	0.08	$500,000	$905	3.7	$	20,965,142	22.8%	6.5
	$10	95,557	$	0.08	$500,000	$905	3.6	$	24,194,510	25.1%	5.8
	$15	95,557	$	0.08	$500,000	$905	3.4	$	27,423,879	27.5%	5.3
	$20	95,557	$	0.08	$500,000	$905	3.2	$	30,653,248	30.0%	4.9
	$25	95,557	$	0.08	$500,000	$905	3.1	$	33,882,616	32.5%	4.6
	$30	95,557	$	0.08	$500,000	$905	3.0	$	37,111,985	35.2%	4.3
	$35	95,557	$	0.08	$500,000	$905	2.8	$	40,341,353	37.9%	4.0
	$40	95,557	$	0.08	$500,000	$905	2.7	$	43,570,722	40.7%	3.8
	$45	95,557	$	0.08	$500,000	$905	2.6	$	46,800,090	43.7%	3.6
	$50	95,557	$	0.08	$500,000	$905	2.5	$	50,029,459	46.8%	3.4
	$0	47,779	$	0.08	$500,000	$905	4.0	$	17,735,773	20.6%	7.3
	$5	47,779	$	0.08	$500,000	$905	3.9	$	19,350,458	21.7%	6.8
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	$15	47,779	$	0.08	$500,000	$905	3.6	$	22,579,826	24.0%	6.1
	$20	47,779	$	0.08	$500,000	$905	3.6	$	24,194,510	25.1%	5.8
	$25	47,779	$	0.08	$500,000	$905	3.5	$	25,809,195	26.3%	5.6
	$30	47,779	$	0.08	$500,000	$905	3.4	$	27,423,879	27.5%	5.3
	$35	47,779	$	0.08	$500,000	$905	3.3	$	29,038,563	28.7%	5.1
	$40	47,779	$	0.08	$500,000	$905	3.2	$	30,653,248	30.0%	4.9
	$45	47,779	$	0.08	$500,000	$905	3.2	$	32,267,932	31.2%	4.8
	$50	47,779	$	0.08	$500,000	$905	3.1	$	33,882,616	32.5%	4.6
	$0	144,784	$	0.25	$500,000	$905	4.0	$	17,735,773	20.6%	7.3
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	$40	144,784	$ 0.25	$500,000	$905	2.3	$ 56,879,635	53.6%	3.1
	$45	144,784	$ 0.25	$500,000	$905	2.2	$ 61,772,617	58.9%	2.9
	$50	144,784	$ 0.25	$500,000	$905	2.1	$ 66,665,600	64.5%	2.7
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	$50	47,779	$ 0.25	$500,000	$905	3.1	$ 33,882,616	32.5%	4.6

BEFORE THE

PUBLIC UTILITY COMMISSION OF OREGON

NW Natural

Exhibit 505 of Barbara Summers

UM 1744
Carbon Emission Reduction Program
Combined Heat & Power (CHP)

Memorandum regarding the Effective Use of Reverse Auctions, Anne Rung, Executive Director, Executive Office of the President, Office of Management and Budget

October 16, 2015

OFFICE OF FEDERAL
PROCUREMENT POLICY

EXECUTIVE OFFICE OF THE PRESIDENT
OFFICE OF MANAGEMENT AND BUDGET
WASHINGTON, D.C. 20503

June 1, 2015

MEMORANDUM FOR CHIEF ACQUISITION OFFICERS
SENIOR PROCUREMENT EXECUTIVES

FROM: Anne E. Rung
Administrator

SUBJECT: Effective Use of Reverse Auctions

This past December, the Office of Federal Procurement Policy (OFPP) issued guidance directing that agencies take a series of actions to foster innovation, increase savings, and improve performance in the acquisition process.[1] For commonly purchased goods and services, these goals will be pursued through category management and a broad set of supporting strategies to achieve better results. Reverse auctions are one of the tools agencies have used in recent years to acquire certain common needs, such as commercial off-the-shelf information technology (IT) hardware and software. In a report published December 9, 2013, the Government Accountability Office (GAO) noted the increased use of reverse auctions at a number of agencies and recommended that OFPP issue guidance to help ensure agencies capture savings and other benefits of this tool.[2] This memorandum reviews the benefits of reverse auctions, offers a set of reminders to help contracting offices maximize the value of this tool, and asks agencies to work with OFPP in identifying and collecting data that can be used to evaluate and improve results.

The value of reverse auctions

A reverse auction is a process for pricing contracts supported by an electronic tool where offerors bid down, as opposed to the traditional auction which requires buyers to submit sequentially higher bids, the main goal of which is to drive prices downward. Offerors are given the opportunity to continually revise their prices during the bidding process until the auction closes. Multiple benefits have been identified in connection with the use of reverse auctions, including the following:

Price reductions. When properly used in combination with other source selection principles, reverse auctions can yield noteworthy savings. GAO notes that the four agencies it studied (Army, Department of Homeland Security (DHS), Department of the Interior, and the

[1] See *Transforming the Marketplace: Simplifying Federal Procurement to Improve Performance, Drive Innovation, and Increase Savings* (December 4, 2014), available at http://www.whitehouse.gov/sites/default/files/omb/procurement/memo/simplifying-federal-procurement-to-improve-performance-drive-innovation-increase-savings.pdf.
[2] See REVERSE AUCTIONS: Guidance Is Needed to Maximize Competition and Achieve Cost Savings (GAO-14-108), http://www.gao.gov/products/GAO-14-108.

Department of Veterans Affairs (VA)) reported approximately 12% in savings from purchases totaling more than $800 million during fiscal year (FY) 2012 for a range of commercial items, including IT, laboratory equipment, furniture, and detection and radiation equipment. The Department of Energy separately reported seeing an average savings of about 14% per contract awarded to provide core supplies and services for its National laboratories. These savings were generally calculated by comparing the agency's independent government cost estimate to the closing price of the reverse auction.

Savings have been reported both through open market purchases (e.g., often for purchase orders awarded under the simplified acquisition threshold (SAT)) and by leveraging existing multiple award contracts. The latter include the Federal Supply Schedules managed by the General Services Administration (GSA) and government-wide acquisition contracts (GWACs), such as the Department of Health and Human Services' Electronic Commodities Store GWAC and DHS's FirstSource contract for IT commodities, which is a total small business set-aside. GSA reports that agencies who conducted reverse auctions against Schedule contracts using its electronic platform, which launched in FY 2013, achieved savings of 19% and more than 23% in FY13 and FY14, respectively.

Enhanced competition. Reverse auctions offer the ability to conduct robust, real-time price competitions. They allow for multiple "rounds of bidding" for continued price reduction. This type of interactive bidding, when it occurs, strengthens competition.

Significant small business participation. GAO reported that 80% of the dollars awarded through the reverse auctions it reviewed from FY 2012 were made to small businesses. A number of agencies have reported continued success in driving dollars to small businesses. For example, agencies have awarded 85% of auctions to small businesses using GSA's reverse auction tool since it was launched in July 2013.

Getting the best results from reverse auctions

As with all procurement tools, effective use of reverse auctions requires careful planning and execution. Contracting officers should consider the following issues to help optimize the results achieved from reverse auctions:

Is the requirement suited for a reverse auction? Reverse auctions are not a one-size-fits-all tool. Reverse auctions are likely to be most effective in a highly competitive marketplace when requirements are steady and relatively simple and might otherwise be acquired using either a sealed bid or achieving best value through "low price technically acceptable" source selection criteria, and result in fixed price agreements. These circumstances would typically exist in acquisitions for commercial items and simple services that often fall under the SAT. As with any procurement, market research must be conducted to understand the marketplace and to determine if it is reasonable to assume that the potential benefits of a reverse auction can be achieved.

Is the agency capturing and reviewing data from prior reverse auctions? A number of reverse auction tools capture prices paid information, as well as offered prices made during the auction. This information has a number of important benefits. In particular, this information can help agencies formulate more accurate government cost estimates, which, in turn, helps to ensure fair and reasonable pricing. Outside of reverse auctions, this cost information (used in conjunction with relevant non-cost information) may help an agency as it looks for more competitive prices for similar items on existing contracts, and reduce overall contract duplication.

GSA's reverse auction tool, which can be used in conjunction with its Schedule contracts, VA's Schedule contracts, Federal Strategic Sourcing Blanket Purchase Agreements (BPAs), agency BPAs against GSA Schedules, and other agencies' contracts (e.g., DHS First Source II), captures detailed (level III) prices paid spending data from past reverse auctions. Agencies can access prices paid information through the Common Acquisition Platform,[3] a tool that GSA has launched to help agencies identify best-in-class contracts issued by GSA and other agencies, best practices, and other information agencies need to reduce the proliferation of duplicative contract vehicles and deliver the best value possible to federal customers and the American people.

To ensure the competition benefits of reverse auctions are being appropriately leveraged, agencies should review any available data on offers received and consider questions such as the following: Is the agency getting more bidders? If the agency is getting a similar number of bidders as it did without using a reverse auction, is it getting interactive bidding? If not, is the transparency of the bids helping to generate lower prices than the government was getting previously? If the agency has previously used a reverse auction and gotten only one bid, has it taken steps that it believes will increase interest in the auction to justify any fees it may be paying to a third party provider?

Is the agency promoting small business participation to the maximum extent practicable? Agencies remain fully responsible for adhering to all applicable small business contracting policies when using reverse auctions. In general, agencies are required to automatically set-aside work for small businesses when the anticipated dollar value is below the SAT. If a determination is made that a small business set-aside is inappropriate, contracting officers must document the reason. For acquisitions above the SAT, contracting officers must set-aside for small businesses when there is a reasonable expectation that offers will be obtained from at least two responsible small business concerns and an award will be made at fair market prices.[4]

When a requirement is set-aside for small business, this information must be conveyed in the solicitation and notice for a reverse auction so that participation in the auction is appropriately limited. In both set-aside and non-set-aside solicitations, the contracting officer must take reasonable steps to ensure that the offerors have access to information regarding the process and any expectations when utilizing reverse auctions, including contact information of the contracting official who will answer questions about the solicitation.[5]

[3] https://hallways.cap.gsa.gov
[4] See FAR 19.502-2 Total small business set-asides.
[5] See FAR 5.102(c)(2).

Has the agency sought feedback from the vendor? While use of reverse auctions in federal contracting has increased in recent years, agency experience with this tool is likely to be more limited than with many other more established practices. Vendor feedback may be particularly helpful as agencies build experience and work to generate robust competition. Accordingly, agencies are encouraged to elicit feedback from auction participants, including experiences with a third party contractor, if one was used to facilitate the competition.[6]

Have the appropriate internal controls been followed? An agency should ensure its contracting staff is carrying out its statutory and regulatory responsibilities, irrespective of whether a third party contractor is used to support the effort. This includes making sure that the contract file is documented[7] with market research results, an independent government cost estimate, vendor quotes, brand name justifications (where applicable), a price reasonableness determination, and documentation that the vendor is a responsible source.

Has the workforce been provided tools, guidance, and/or training? Agencies must ensure that members of the acquisition workforce are trained and are familiar with any agency-specific policies and procedures that govern the use of reverse auctions. Online continuous learning modules, CLC 031 – Reverse Auctioning and FAC 052 – The GSA Reverse Auction Platform, are available from the Defense Acquisition University (DAU) and the Federal Acquisition Institute (FAI).[8] These courses provides a basic introduction to the process of using reverse auctions.

Does the agency regularly review its reverse auction practices and policies? Like other acquisition tools, agencies should be evaluating their experiences with reverse auctions and the effectiveness of existing practices and policies as part of its procurement management reviews so that refinements can be made as necessary. To support these efforts, OFPP intends to convene a working group to review needs for standardized data collection and other matters (see next steps below).

Additional considerations when using a third party contractor

When agencies decide to contract with a vendor to conduct reverse auctions (hereinafter referred to as a "third party contractor,") agencies must consider the following additional issues:

Fees. Contracting officers should negotiate a fee structure with a private sector service provider that provides the best value to the government. There are multiple ways in which fees might be charged when a third party contractor is used. The cost to conduct a reverse auction may be a percentage of the transaction, a percentage of the savings, or a flat fee. Whatever the arrangement, agencies must make a determination before awarding a contract with a third party contractor that the fee structure represents a fair and reasonable cost for the reverse auction

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[6] For general guidance on the use of vendor feedback surveys to target opportunities for improved acquisition practices, agencies may wish to consider *Acquisition 360-Improving the Acquisition Process through Timely Feedback from External and Internal Stakeholders* (March 18, 2015) available at https://www.whitehouse.gov/sites/default/files/omb/procurement/memo/acquisition-360-improving-acquisition-process-timely-feedback-external-internal-stakeholders.pdf.
[7] See, FAR Subpart 4.8.
[8] http://icatalog.dau.mil/onlinecatalog/courses.aspx?crs_id=440

service. In addition, fees should be considered in evaluating whether the price of the product or service (including any additional fees for use of another agency's existing contract) is fair and reasonable. Anticipated cost savings should be taken into account in determining the reasonableness of the fee.

In order to maximize competition and small business participation, agencies are encouraged to cover the costs of vendor participation and avoid fee arrangements where vendors must pay to participate in the agency's reverse auction.

Government contracting official responsibilities. Agencies must take additional steps to ensure that the selected third party contractor provides a "seller-neutral" marketplace. The agency remains ultimately responsible for ensuring that third party contractors do not perform inherently governmental functions and that processes are compliant with all procurement laws and regulations, including those associated with protecting the integrity of competition, reviewing past performance, providing appropriate notice of the reverse auction, establishing terms of participation and the basis for source selection, securing proprietary vendor information, and facilitating communications between the agency and vendors during the course of an auction. Agencies should ensure that no contractors are excluded from bidding in an auction by a third party contractor. Only an agency official may exclude a bidder from participating in an auction.

Contract data information. Any information used in a reverse auction conducted by a third party contractor is the property of the Federal Government and should be provided to the agency on a regular basis based on the agreement between the agency and the third party contractor. These data will be used in support of government-wide efforts to reduce duplication and create further savings.

Next steps

To maximize the value of reverse auctions and ensure practices are effective and meeting their intended purposes, OFPP seeks to work with agencies to identify the essential management data points (e.g., price paid for item, fees paid (if any), number of bidders, and level of interactive bidding) and mechanisms for collecting and aggregating information in a manner that leverages technology and avoids the need for manual collection. As explained above, electronic reverse auction tools typically allow agencies to maintain documentation of each auction online, creating a virtual library of prices paid data that is a key component of category management and can be useful in developing better price estimates and purchasing strategies for future requirements. Similarly, terms and conditions can be stored in an easily reusable format for recurring requirements, saving valuable time.

Accordingly, agencies that have used reverse auction tools (either directly or with the assistance of a third party contractor) are asked to provide points of contact to Susan Minson (e-mail: sminson@omb.eop.gov or 202-395-6810) no later than July 10, 2015. As part of this process, OFPP will work with agencies to review methodologies for calculating savings.

Please remind your acquisition workforce of the points and best practices outlined in this memorandum and encourage them to take the online training accessible through FAI and DAU. For your awareness, as a further step, the Federal Acquisition Regulatory Council will open a case to develop coverage on the use of reverse auctions in the Federal Acquisition Regulation and will address the guidance in this memorandum, as appropriate.

Any questions should be directed to Ms. Minson. Thank you for your attention to this guidance.